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The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme

Department	: Mathematics
Level	: 05
Name of the Examination	: Final Examination
Course Code and Title	: **ADU5308**- Graph Theory
Academic Year	: 2020/2021
Date	: 27-03-2022
Time	: 1.30 p.m. – 3.30 p.m.
Duration	: 2 Hours

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of 06 questions in 05 pages.
3. Answer any 04 questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Draw fully labelled graphs where necessary.
6. Having any unauthorized documents/ mobile phones in your possession is a punishable offense.
7. Use blue or black ink to answer the questions.
8. Circle the number of the questions you answered in the front cover of your answer script.
9. Clearly state your index number in your answer script.

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Applied Mathematics– Level 05
ADU5308 – Graph Theory

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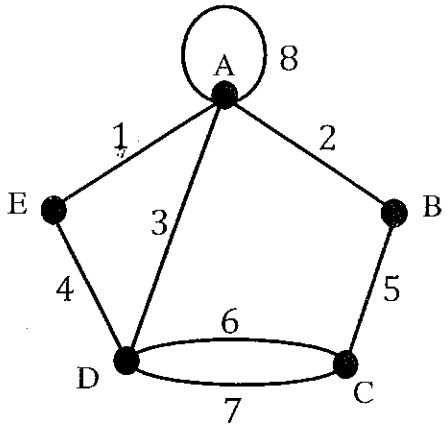
ANSWER FOUR QUESTIONS ONLY.

1.

a) Write the adjacency list, the incidence matric and the adjacency matric of the following graph.



b) Determine whether the graph with given degree sequence exists or not. If exists draw it, otherwise explain why it does not exist.

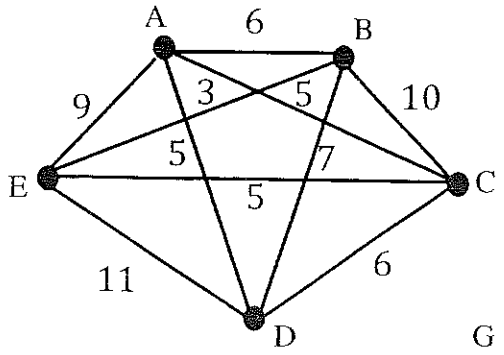
I. 6 vertices of degrees 1, 1, 2, 3, 4 and 4.

II. 4 vertices of degrees 2, 2, 3 and 3.

c)

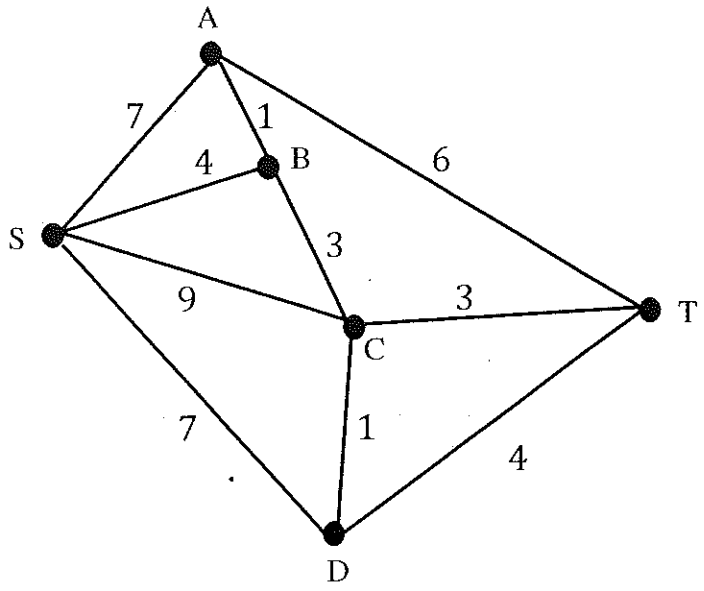
I. Define a Hamiltonian cycle of a graph G.

II. Five cities A, B, C, D and E, and the distance between the cities are given in the following weighted graph G. Solve the Traveling Salesman problem.

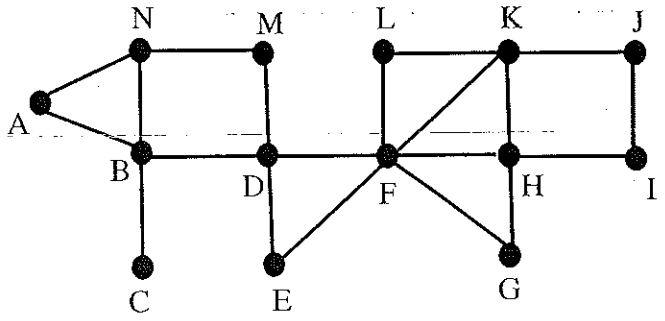


2.

a) Find the shortest path from S to T of the following weighted graph.



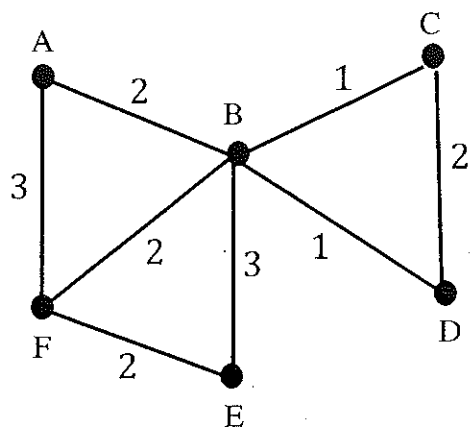
b) Use breath first search algorithm to find the spanning tree for the following graph G, choosing 'F' as the root.



c) Rewrite the mathematical expression $(3x^4 + 10y) - 2(z^3 - 6)$ using " $*$ " for multiplication and " $\uparrow$ " for exponentiation. Then construct the binary rooted tree which represents this expression.

3.

a) Use Kruskal's Greedy algorithm to find a minimum weighted spanning tree for the following weighted graph:



Verify the above result using Prim's Greedy algorithm, by starting with the vertex B.

b) Define the planer representation of a graph G.

I. Is $K_{2,4}$ planer? Justify your answer.

II. Is K_5 planer? Justify your answer.

c) Draw the dual graph of W_4. Is it self-dual? Justify your answer.

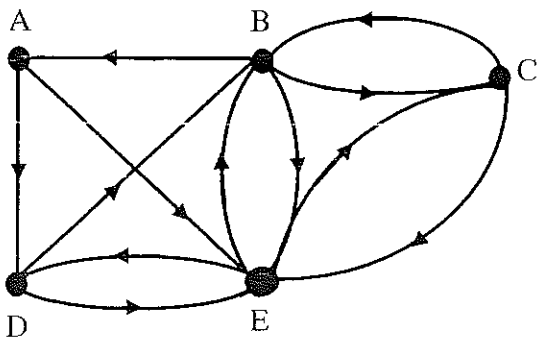
4.

a) Consider the following digraph $D = (V, A)$:

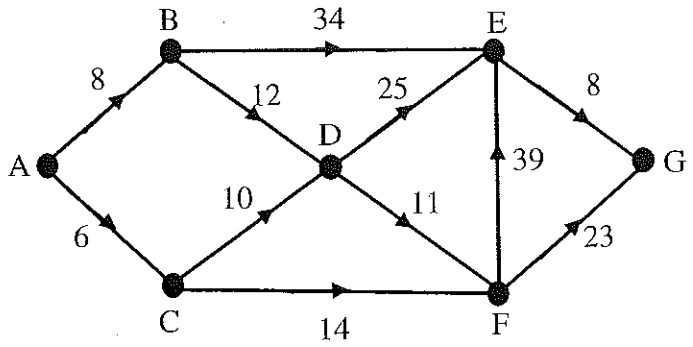
I. Find $d(\mathrm{A}, u)$ for all $u \in V$.

II. Find $d(u, \mathrm{E})$ for all $u \in V$.

III. Determine whether it is strongly connected or not. Justify your answer.



b) The following digraph represents construction of a complete guesthouse where A-represents the beginning of the job and G-represent the completion of the job. Find the critical path and the minimum time required to build the complete guesthouse.

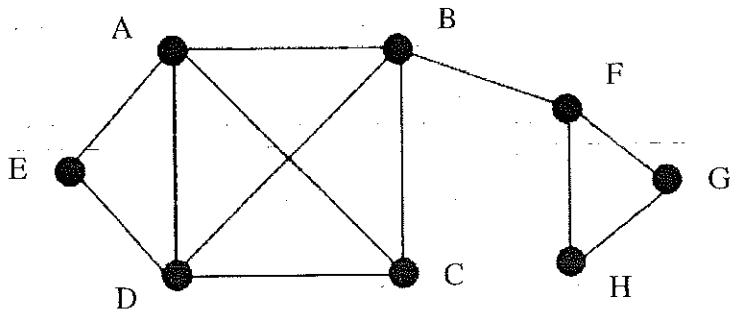


5.

a) Consider the following graph G:

I. Find all the cut points, bridges and blocks of the graph G.

II. Define the Block graph B(G) and Cut point graph C(G) of a graph G. Obtain the corresponding Block graph and Cut point graph for the following graph G.



G

b) Suppose that three boys x, y, z know five girls a, b, c, d, e as given in the table.

Boy	Girls known by a boy
x	a c e
y	a b c
z	b d

I. Draw the bipartite graph corresponding to this table of relationships.

II. Check the marriage condition for this problem.

6.

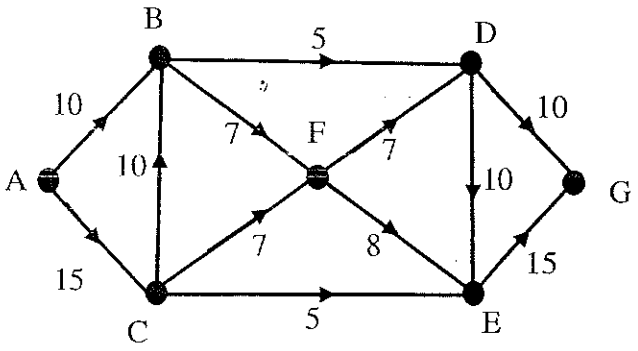
a) Consider the family $\Im = (S_1, S_2, S_3, S_4)$ of subsets of E = {1, 2, 3, 4, 5, 6}, where, $S_1 = \{1, 2, 6\}$, $S_2 = S_3 = \{1, 3, 4\}$ and $S_4 = \{5, 6\}$.

I. Obtain the corresponding incident matrix.
II. Find the term rank.
III. Verify Konig – Egervacy theorem.
IV. Find all traversals of the above family if exist.

b) State Max-flow, min-cut theorem.

I. Verify Max-flow min-cut theorem for the following Network N.

II. Define a saturated arc and write down the saturated arcs for the Network N.



N