00176

The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme

Department	**: Mathematics**
Level	**: 5**
Name of the Examination	**: Final Examination**
Course Code and Title	**: ADU5319 – Design and Analysis of Experiments**
Academic Year	**: 2020/2021**
Date	**: 19.03.2022**
Time	**: 1.30 p.m. – 3.30 p.m.**
Duration	**: 2 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **6 questions in 6 pages**.
3. Answer **4** questions only. All questions carry equal marks.
4. Answers for each question should commence from a new page.
5. The table of critical values for the F distribution corresponding to a 5% significance level is attached at the end.
6. Involvement in any activity that is considered as an exam offense will lead to punishment.
7. Use blue or black ink to answer the questions.
8. Clearly write your index number on each page of your answer script.

1). To compare the effects of three catalysts (*A, B, C)* on the time taken for a chemical reaction to start, a researcher prepared 15 chemical samples each of 10ml and randomly divided into three groups of 5 each. The times to start the reaction (measured to the nearest minute) after adding 1*mg* of a randomly selected catalyst to each chemical sample in the three groups is given in the table.

Catalyst	Time to start (minutes)				
A	12	11	13	12	14
B	10	10	9	9	8
C	15	14	15	14	15

Total corrected sum of squares = 82.9333

i) Define the following terms and identify each, in relation to this study.

a) replicate

b) Experimental unit

c) Treatment

ii) The researcher plans to analyze the data using an analysis of variance (ANOVA) approach. Clearly describing your notation, write down the model equation for the time to start the reaction measured on a randomly selected chemical sample.

iii) Write down the null and the alternative hypotheses that you would examine to find out whether there is a significant difference in the effects of the three catalysts on the time taken for reaction to start.

iv) Construct the complete analysis of variance (ANOVA) table that can be used to test the validity of the hypotheses stated in part (iii).

v) Using the critical values from the F-distribution table (see Appendix), and the ANOVA table constructed, test the hypotheses stated in part (iii) and clearly state your findings.

2). A researcher is interested in comparing the effects of three fertilizers ($F1$, $F2$, $F3$) on the dried weights of a certain medicinal plant species, at the age of four weeks. Furthermore, the researcher wishes to study two fertilizer application methods ($M1$, M2) with each of the three fertilizers. One week old plants grown in different pots with the same soil condition are available for this study. Normally, fertilizer is applied to one-week old plant. The effects are to be compared when the fertilizer is added to one-week old plants.

i) If the researcher plans to use a factorial design, explain how he should decide on the treatment combination to be applied for a chosen plant.

ii) Suppose the researcher plans to use four replicates for each experimental condition. How many plants will the researcher need for this study?

iii) The sample means of the four replicants were computed from the observations collected under each of the experimental conditions described in the table.

Type of fertilizer	Method of application	Mean of the four replicates
$F1$	$M1$	3.4
$F1$	$M2$	3.8
$F2$	$M1$	5.2
$F2$	$M2$	6.1
$F3$	$M1$	3.0
$F3$	$M2$	2.9

Construct a suitable plot that can be used to examine whether the method of application that gives the optimal dried weight depends on the type of fertilizer used.

iv) Clearly state what you conclude from the plot constructed in part (ii).

v) Write down all the sources of variation that you would consider for an Analysis of Variance (ANOVA) table for this study and give the degrees of freedom for each source of variation you mentioned.

vi) A researcher found that the sum of squares corresponding to the error term is 12.04. Estimate the random variation and explain what it measures in relation to this study.

3). The rats used in a study to compare the effects of three diets (*D1, D2, D3)* on the weight gain belonged to three age groups (juvenile, young and adult). Eighteen rats from each age group, were randomly divided into three groups of six rats in each, and one of the diets was assigned to each group. The rats were kept separately, and the weight gains (in grams) after three months were measured.

i) Define the following terms and identify each, in relation to this study.

a) Response variable

b) Blocking variable

ii) State the design used in this study.

iii) Clearly describing your notation, write down the model equation for the weight gain of a randomly selected rat.

iv) The following summary statistics were computed by separating the data according to the three diet groups and according to the three age groups.

Description	Diet group			Age group		
	D1	*D2*	*D3*	Juvenile	Young	adult
Sum of the observations	23.9	24.4	29.9	32.6	25.7	19.9
Sum of the squared observations	100.65	104.64	155.03	178.76	111.13	70.43

Assume that the effects of the diets (if any) on the weight gain of rats are the same across all age groups.

Construct an analysis of variance table that can be used to test whether the effects of the three diets on the weight gains of the rats are all equal or not.

v) Using a 5% significance level, test whether there is a significant difference in the effects of the diets on the mean weight gains of rats.

4). In a study to compare three metals with regards to the resistance for corrosion, a researcher recorded the time (measured in months) to first signs of corrosion from the start of the study, in 15 metal rods kept in similar environmental conditions. The data collected are presented in the following table.

Metal of the rod	Time to first signs of corrosion (months)
A	9, 6, 6, 7, 7
B	9, 10, 11, 9, 11
C	12, 11, 12, 13, 12

i) Calculate the total corrected sum of squares and the corresponding degrees of freedom.

ii) Clearly state the assumptions one must make in order to test whether there is a significant difference between the corrosion times of different metal rods, using an analysis of variance table.

iii) Suppose metal *A* is of alloy type I and metals *B* and *C* are of alloy type II.
The researcher wants to compare whether there is a significant difference between the corrosion times of

a) metals of the two different alloy types

b) metals *B* and *C*

Clearly describing your notation, write down contrasts for each of the researcher's objectives described in (a) and (b).

iv) Are the contrasts(a) and (b) stated in part (iii) orthogonal comparisons? Give reasons for your answer.

v) Calculate the least significant difference (LSD) and determine whether the mean corrosion times of metals *B* and *C* are significantly different or not, using a 5% significance level. Clearly state your findings. (You may calculate critical values for the student t-distribution with degrees of freedom *v* by taking the square root of the *F* table value with numerator degrees of freedom 1 and denominator degrees of freedom *v.)*

5). A quality inspector wanted to compare the strengths of thread received from three suppliers (*S*1, *S*2, *S*3), by measuring the time (in hours) until the first thread break. The threads are to be tested on three machines (*M*1, *M*2, *M*3). The quality inspector employed three machine operators to operate the machines as randomly assigned by the quality inspector using the proposed thread. The times for the first thread break, from the start of the machine are given within the bracket of each cell in the table below.

Machine	Operator1	Operator2	Operator3
M1	S1 (4.5)	S2 (3.5)	S3 (2.0)
M2	S2 (5.0)	S3 (4.5)	S1 (4.5)
M3	S3 (4.0)	S1 (5.0)	S2 (3.0)

i) Name the design used in this study.

ii) Suggest another suitable assignment order of threads and machines, which will satisfy the design proposed in part (i).

iii) Construct an appropriate analysis of variance table.

iv) Test whether there is a significant difference in the mean times for the first thread break of the threads supplied by the three suppliers using a 5% significance level. Clearly state your findings.

6). In a study to compare the yields of four varieties of paddy, by measuring the yield per plot (in grams). The researcher has access to plots in three different locations. The researcher suspects differences in the soil conditions of the locations. At each location, the researcher has access to four plots that are far apart.

i) Suppose the researcher seeks your advice to design this study. Briefly describe how you advise.

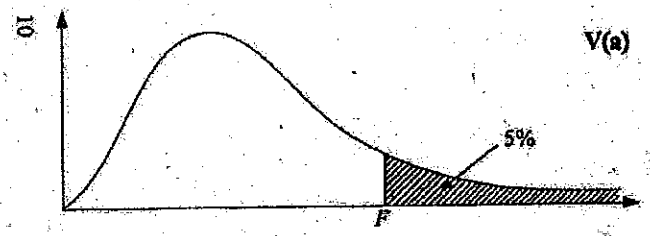
ii) Describe one advantage of blocking in a designed experiment.

iii) Part of the analysis of variance (ANOVA) table constructed from the data collected are given below. Complete the ANOVA table by filling in the missing values.

Source of variation	Sum of squares	Degrees of freedom	Mean sum of squares	*F* ratio
variety	177.26	***	***	***
location	9.26	***	***	***
residual	***	***	***	
Total	198.95	***		

iv) Test whether there is a significant difference in the locations used in this study, using a 5% significance level. Clearly state the findings.

Appendix



5 ප්‍රතිශතයට අනුරූප F ව්‍යාප්තිය
F பரம்பலின் 5 சதவீத புள்ளிகள்
5 percent points of the F distribution

n_2 =	n_1 = 1	2	3	4	5	6	7	8	10	12	24
2	18.5	19.0	19.20	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.79	8.74	8.64
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	5.96	5.91	5.77
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.74	4.68	4.53
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.06	4.00	3.84
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.64	3.57	3.41
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.35	3.28	3.12
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.14	3.07	2.90
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	2.98	2.91	2.74
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.75	2.69	2.51
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.54	2.48	2.29
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.35	2.28	2.08
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.25	2.18	1.98
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.16	2.09	1.89
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.08	2.00	1.79
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	1.99	1.92	1.70