00092

The Open University of Sri Lanka

Advanced Certificates in Science – Level 2 Part 1

Final Examination – 2020/2021

Duration: Three (03) hours

MHF2519 - Mathematics 1–Paper I

Date: 7th December 2021 **Time: 09.30 am – 12.30 pm**

Instructions

- **You are allowed to use non-programmable calculators.**
- **Access to mobile phones during the test period is prohibited.**
- **Answer five (05) questions including at least one question from part B.**

Part A – Algebra

Q1. (a). Find the domain and the range of the following functions.

(i). $f(x) = \dfrac{x-1}{x+1}$

(ii). $h(t) = \sqrt{t^2+1}$

(b). (i). Find the domain and the range of the function, $f(x) = 3x^2 - 2$.

(ii). Sketch the graph of $f(x)$, and mark the domain and range on it.

(c). Given that $f(x) = 2x - 1$ and $g(x) = 3x^2 + 2x - 1$. Find

(i). $f(g(x))$.

(ii). $g(f(x))$.

Q2. (a). Simplify the following expressions with positive indices.

(i). $\left(\frac{125}{64}\right)^{-\frac{1}{3}} \times \left(\sqrt[5]{32}\right)^3 \times 3^0$

(ii). $\sqrt[3]{343x^{3/2}} \div x$

(b). Solve the following index equation for x.

(i). $\frac{3^{1+x}}{27^{3+x}} = 9$

(ii). $6^x \times 216^{3x} = 36^2$

(iii). $3^{2x+1} - 28(3^x) + 9 = 0$

(c). Sketch the graph of $y = e^x$ and $y = e^{-x}$ in a same plane.

Q3. (a). Given that $\log_7 2 = \alpha, \log_7 3 = \beta$ *and* $\log_7 5 = \gamma$, express the following expressions in terms of α, β *and* γ.

(i). $\log_7 6$

(ii). $\log_7 75$

(iii). $\log_7 \frac{15}{2}$

(b). Solve the following logarithmic equations for x.

(i). $\log_2(x+2) + \log_2(3) = \log_2(27)$

(ii). $\log_2(x) + 6\log_x(2) - 5 = 0$

(c). The change of base formula is given by

$$\log_b x = \frac{\log_a x}{\log_a b}.$$

By using the above formula evaluate $\log_4 8$.

Q4. (a). Express the function of $f(x) = x^2 - 6x + 7$ in completed square form. Find its,

(i). the zeros (roots)

(ii). the line of symmetry

(iii). coordinate of the vertex and sketch the graph and mark all findings.

(b). Find the condition of k for the quadratic equation $x^2 - kx + k = 0$, to have

(i). repeated real roots

(ii). two distinct real roots

(iii). no real roots

Q5. (a). (i). α and β are roots of the equation $x^2 - px + q = 0$. Find the equation whose roots are α^3 and β^3.

(ii) If one root of the equation $ax^2 + bx + c = 0$ is three times of the other, then show that $3b^2 = 16ac$.

(b). Using the Principle of mathematical Induction, prove that $n^3 + 6n^2 + 8n$ multiple of 3 for all positive integers n.

Q6. (a) The remainders when $p(x) = ax^3 + bx + c$ is divided by $(x + 1)$, $(x - 1)$ and $(x - 2)$ are 4, 0 and 4 respectively. Find the values of a, b, c and determine all linear factors of $p(x)$.

(b). Use the factor theorem to find the real roots of the following polynomial. $p(x) = x^4 + 4x^3 - x^2 - 16x - 12$

Part B – Coordinate Geometry

Q7. The vertices of a triangle are $A(1,0)$, $B(2,3)$, $C(5,2)$.

(a). Find the equation of the line AC.

(b). Find the equation of the line through B perpendicular to AC.

(c). Determine whether the triangle is right angled. (Hint: find the gradients of the sides AB and BC)

(d). Find the angle between AB and AC.

(e). Calculate the area of the triangle

Q8 (a). If one end of a diameter of a circle, $x^2 + y^2 - 2x - 2y - 3 = 0$ is $(2, 3)$ find the coordinate of the other end.

(b). Find the equation of the circle passing through the points (2, -1) and (1, 1) and having its center on the line $y - 3x + 7 = 0$.

END.