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The Open University of Sri Lanka

Advanced Certificates in Science – Level 2 Part 1

Final Examination – 2020/2021

Duration: Three (03) hours

MHF2519 - Mathematics 1–Paper II

Date: 8th December 2021 **Time: 9.30 am – 12.30 pm**

Instructions

- **You are allowed to use non-programmable calculators.**
- **Access to mobile phones during the test period is prohibited.**
- **Answer Six (06) questions including at least two questions from each Part.**

Part A – Trigonometry

Q1 **(a).** A bicycle with tires 90 cm in diameter is travelling at speed of 25 kmh^{-1}. Find the angular velocity of the tire in radians per second

(b). From the top of a building 30 m high, angles of depression of two objects at ground are observed as 60^0 and 45^0 respectively. If the top of the building and the two objects are in the same vertical plane, then find the distance between two objects.

(c). **(i).** If $5\tan\alpha = 4$, find the value of $\dfrac{5\sin\alpha - 3\cos\alpha}{\sin\alpha + 2\cos\alpha}$

(ii). Evaluate, without using calculators, $\cos(-765^0)$, $\sin\left(\frac{8\pi}{3}\right)$, $\tan\left(\frac{9\pi}{4}\right)$

Q2 (a). Prove the following identity.

$$\cot\left(\frac{\pi}{4}+\theta\right)\cot\left(\frac{\pi}{4}-\theta\right)=1$$

(b). Given that, $a(\tan\theta+\cot\theta)=2$, $b(\tan 2\theta+\cot 2\theta)=2$.
Show that $a=\sin 2\theta$ and $b^2=4a^2(1-a^2)$.

(c). If $\cos 2\theta=\tan^2 x$, then show that $\cos 2x=\tan^2\theta$

Q3 (a). Using half angle formulae show that,

$$\cos\left(22\frac{1}{2}\right)^0=\frac{\sqrt{2+\sqrt{2}}}{2}$$

(b). Prove that $\cos(A+B)\cos(A-B)=\cos^2 A-\sin^2 B$.

(c). Using $t=\tan\left(\frac{x}{2}\right)$ find the values of $\tan x$ as a rational function of t. **Hence** deduce $\sin x$ and $\cos x$ in terms of t.

Part B – Statics

Q4. (a). When two forces are acting perpendicular to each other their resultant is $\sqrt{5}$ N. When those two forces are acting inclined at an angle 60^0 each other the resultant is $\sqrt{2}$ N. Find two forces.

(b). The angle between forces F_1 and F_2 is 112^0. The measure of the angle between their resultant and F_2 is 56^0. If the magnitude of F_1 is 28 N, find the magnitude of F_2.

Q5 $ABCDEF$ is a regular hexagon of side $2a$ m. Forces of magnitude $P, 2P, Q, Q, 2P$ and P newtons act along the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DE}$, $\overrightarrow{EF}$ and $\overrightarrow{FA}$ respectively. If the resultant force of the system act through E, prove that it will act through C also. Find the magnitude of the resultant and the distance from B where its meets AB.

Q6 (a). In triangle OAB, $\underline{a}$ and $\underline{b}$ are the position vectors of A and B. D is the midpoint of OA and C is the point on AB, such that $AC:CB = 2:3$. Express $\overrightarrow{OC}, \overrightarrow{BD}$ and $\overrightarrow{CD}$ in terms of $\underline{a}$ and $\underline{b}$.

(b). The position vectors of the points P and Q are $\underline{p}$ and $\underline{q}$ respectively. PQ divides internally at R and externally at S so that $PR:RQ = PS:QS = m:1$. Show that

$$\overrightarrow{RS} = \frac{2m(\underline{p} - \underline{q})}{1 - m^2}.$$

Part C – Dynamics

Q7 A particle moves with uniform acceleration f, in a horizontal line. At time t, displacement is a and at time $2t$ displacement is $(a + b)$. By using equation of motion find,

(a). the acceleration f

(b). the speed of the particle after time $2t$

(c). displacement after time $3t$.

Q8 (a). A particle is projected vertical upwards with a speed of 40 ms^{-1}. Find the speed after 5 s and the greatest height reach by the particle. ($g = 10\ ms^{-2}$)

(b). A train starts from rest at station A and moves with uniform acceleration for 60s until it reaches a speed of 30 ms^{-1}. It travels at this constant speed for T seconds and then deceleration uniformly for 1.2 km, coming to rest at station B which is 14.1 km from station A.

(i). Sketch a speed-time graph for the journey.

(ii). Calculate the deceleration of the train

(iii). Calculate the value of T.

(iv). Calculate the total time for the journey.

Q9 A particle is projected with initial velocity of v at an angle $\propto$ above the horizontal. If the greatest height of the particle is h and the horizontal range is a, show that

(a). $\propto = \tan^{-1}\left(\frac{4h}{a}\right)$

(b). $v = \left[2g\left(h + \frac{a^2}{16h}\right)\right]^{\frac{1}{2}}$.

(c). When $h = 2a$ evaluate the values of $\propto$ and v.

(END).