00081

The Open University of Sri Lanka

Advanced Certificates in Science – Level 2 Part 2

Final Examination – 2020/2021

Duration: Three (03) hours

MHF2521 - Mathematics 3–Paper I

Date: 7th December 2021 **Time: 01.30 pm – 04.30 pm**

Instructions

- **You are allowed to use non-programmable calculators.**
- **Access to mobile phones during the test period is prohibited.**
- **Answer five (05) questions including at least one question from part B.**

Part A – Calculus

Q1. (a). Find the limits of the following functions.

(i). $\lim_{x \to 4} \frac{x^2 - 7}{(x+2)(x^2 - 1)}$

(ii). $\lim_{x \to 4} \frac{x - \sqrt{3x + 4}}{4 - x}$

(iii). $\lim_{\theta \to 0} \frac{(1 - \cos\theta + \sin\theta)}{(1 - \cos\theta - \sin\theta)}$

(iv). $\lim_{x \to 0} \frac{\sin x^o}{x}$, where x^o represents degree x.

(b). Differentiate the following functions using the first principles.

(i). $y = \frac{1}{1 - 2x}$

(ii). $y = \sin x$

Q2 (a). Differentiate the following functions and simplify your answer.

(i). $y = (1-x)\sqrt{1+x^2}$

(ii). $y = x^3(4-x)^{\frac{1}{2}}$

(iii). $y = \dfrac{\sqrt{x}}{3x+1}$

(iv). $y = \dfrac{(x^2+1)^8}{x^5}$

(b). The amount of air in a balloon at any time t is given by

$$V(t) = \frac{6(\sqrt[3]{t})}{4t+1}.$$

At t=8, determine if the balloon is being filled with air or being drained of air.

Q3 (a) Differentiate the following trigonometric functions and simplify the answer.

(i). $y = \dfrac{\sin x - \cos x}{\sin x + \cos x}$

(ii). $y = \sec^2\dfrac{x}{2} + \operatorname{cosec}^2\dfrac{x}{2}$ (Give the answer in full angles of x)

(b) If $y = \cos^{-1}\left(\dfrac{3+5\cos x}{5+3\cos x}\right)$ prove that $\dfrac{dy}{dx} = \dfrac{4}{5+3\cos x}$.

Q4 (a). Differentiate the following functions with respect to x.

(i). $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

(ii). $y = \dfrac{x^2}{2^x}$

(iii). $y = \ln\left(\dfrac{\sqrt{x}}{x^2+4}\right)$

(b). If $y = a\cos(\log x) + b\sin(\log x)$, prove that

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$$

(c). The graph of the curve is $y = \frac{x+1}{x^2+x+1}$. Find all points on the curve where the tangent line is horizontal.

Q5 (a). If $x = \frac{2t}{1+t^2}$ and $y = \frac{1-t^2}{1+t^2}$, where t is a parameter, find the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of t.

(b). Find the equation of the tangent line to the function of $f(x) = 1 - \frac{1}{x} + \frac{2}{\sqrt{x}}$

at the point $(4, \frac{7}{4})$.

(c). If the position of an object moving along a straight line is given by $s(t) = 3t^5 - 5t^3 - 7 = 0$ find the object's velocity $v(t)$ and acceleration $a(t)$. Find all values of t when the acceleration is zero.

Q6 (a). Find the turning points of the function of $y = x^3 - 6x^2$. Considering only the behavior of the first derivative, identify nature of each point. Find the point of inflection using the second derivative.

(b). Evaluate the following integrals.

(i). $\int \frac{7}{(2-3x)^8} dx$

(ii). $\int [(e^x - x^e) + \ln(2x+1)] dx$

(iii). $\int \frac{x^3 - 4x + 3}{x - 2} dx$

(iv). $\int \left[\frac{\tan x}{\sin x \cos x} + \frac{1}{16 + x^2}\right] dx$

Part B – Coordinate Geometry

Q7 (a). Given that the area of the triangle made by the straight lines $y = m_1x + c_1$ $y = m_2x + c_2$ and $x = 0$ is $\frac{(c_1-c_2)^2}{2|m_1-m_2|}$. Hence, evaluate the area of the triangle made by the straight lines $y = 2x + 3$, $y = -x + 3$ and $y = x + 1$.

(b). Show that the straight line, $2x - 3y + 26 = 0$ is tangent to the circle

$x^2 + y^2 - 4x + 6y - 104 = 0$. Find the equation of the diameter, passing through the tangent point.

Q8 (a). If two circles are cut orthogonally find the condition of the equation to be satisfied.

(b). Find the equation of the circle which passes through the point (1, 2) and cuts orthogonally each of the circles $x^2 + y^2 = 9$ and

$x^2 + y^2 - 2x + 8y - 7 = 0$.

END.