The Open University of Sri Lanka

Advanced Certificates in Science – Level 2 Part 2

Final Examination – 2020/2021

Duration: Three (03) hours

MHF2521 - Mathematics 3–Paper II

Date: 8th December 2021 **Time: 1.30 pm – 4.30 pm**

Instructions

- **You are allowed to use non-programmable calculators.**
- **Access to mobile phones during the test period is prohibited.**
- **Answer five (05) questions including at least one question from each part.**

Part A - Algebra

Q1 **(a).** If $f(r) = \frac{1}{r^2}$ $(r \neq 0)$, then show that $f(r+1) - f(r) = -\frac{2r+1}{r^2(r+1)^2}$.

Hence, using the method of differences find the sum of the first n terms of the series:

$$\frac{3}{1^2 \cdot 2^2} + \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \cdots \ldots\ldots\ldots\ldots\ldots .$$

Is the above series convergent? Justify your answer

(b). Using the Principle of Mathematical Induction, Prove that

$$1^3 + 2^3 + \cdots + n^3 = \frac{n^2}{4}(n+1)^2.$$

Q2 (a). Use the method of partial fractions and find the sum of first n terms of the series:

$$\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}$$

Is the above series convergent? Justify your answer.

(b). Evaluate the value of the following series

$$\sum_{n=1}^{10} \frac{3}{(-2)^{n-1}}$$

Find the sum up to infinity of this series.

Part B – Statics

Q3 (a). QRS is a triangle, Q and S, have position vectors $\underline{i} + 4\underline{j}$ and $5\underline{i}$ respectively. C is the centroid this position vector is $2\underline{i} - \underline{j}$. Find the position vector of vertex R.

(b). Find the vector equation passing through the points A (4, -2, 3) and B (2, 1, -3). Find direction cosines of AB.

(c). ABC is a triangle. The position vectors of vertexes A, B and C are

$\underline{a} = 3\underline{i} - 2\underline{j} + \underline{k}$, $\underline{b} = 3\underline{i} + 2\underline{k}$, $\underline{c} = \underline{i} + 2\underline{j} + 5\underline{k}$ respectively. Find the angle between BD and AC, where D is the mid-point of AC.

Q4 (a). If $\underline{A} = 3\underline{i} - 2\underline{j} - \underline{k}$ and $\underline{B} = \underline{i} + 3\underline{j} + 7\underline{k}$, find the vector perpendicular to both $\underline{A}$ and $\underline{B}$ with the magnitude of 6.

(b). Find the vector equation of the cartesian line denoted by $2x - 3y + 12 = 0$.

Q5 (a). Find the vector equation of the line parallel to the $\underline{m} = 4\underline{i} + 5\underline{j}$ and passing through the point A (2, 3). **Hence**, find the parametric equations of the line and the cartesian equation.

(b). In a triangle, ABC vertexes are A (2, 3), B (4, -5) and C (-3, 2). Calculate the area of the triangle ABC.

Q6 (a). Find a unit vector which is perpendicular to the plane of the two vectors $\underline{a} = 4\underline{i} + 3\underline{j} - \underline{k}$ and $\underline{b} = 2\underline{i} - 6\underline{j} - 3\underline{k}$.

(b). For any triangle ABC, prove that $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ by using vectors.

Part C – Dynamics

Q7 A particle of mass m is attached to P at one end of a light inelastic string of length l and other end attached to A. The second string with same length is attached at point P and other end is attached to point B. B located vertically below with length h ($< 2l$) from A. Particle moves in a horizontal circular path with angular velocity of ω. Find tension of two strings. Show that, if $\omega \geq \sqrt{\frac{2g}{h}}$ both strings will be taut.

Q8 One end of a light inextensible string of length l is attached to a fixed-point A and the other end to a particle B of mass m which is hanging freely at rest. The particle is then projected horizontally with velocity $\sqrt{\frac{7gl}{2}}$. Calculate the height of B above A when the string goes slack. When the string is horizontal, find the velocity of the particle and tension of the string.

Q9 A mass $2m$ rests on a smooth horizontal table and is connected to a mass m by a light inextensible string passing through a small ring fixed at a height h above the table. If the mass 2m is made to describe a circle of radius $\frac{h}{2}$ having its center on the table vertically below the ring, show that the time it takes to describe the circles ones is

$$2\pi\left[\frac{h\sqrt{5}}{g}\right]^{\frac{1}{2}}.$$

END.