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The Open University of Sri Lanka

Advanced Certificates in Science – Level 2 Part 2

Final Examination – 2020/2021

Duration: Three (03) hours

MHF2522 - Mathematics 4—Paper I

Date: 27th December 2021 **Time: 1.30 pm –4.30 pm**

<u>Instructions</u>

You are allowed to use non-programmable calculators. Access to mobile phones during the test period is prohibited.

Answer five (05) questions including at least one question from each part.

Part A – Algebra

(1)(a) Express the following complex numbers of the form r, θ. Write down the modulus and the argument for each number.

(i) $\dfrac{1+7i}{(2-i)^2}$ (ii) $\dfrac{\left(1+i\sqrt{3}\right)^2}{4i\left(1-i\sqrt{3}\right)}$

(b) Show that,

$$\frac{\cos\alpha + i\sin\alpha}{\cos\beta + \sin\beta} = \cos(\alpha-\beta) + i\sin(\alpha-\beta)$$

Let $z_1 = -1 + i$ and $z_2 = 1 + i\sqrt{3}$

Find the real part and the imaginary part of the form where, $\frac{z_1}{z_2}$.

Express each of z_1 and z_2 in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $0 < \theta < \pi$.

Deduce that $\cos\frac{5\pi}{12} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$.

(2) Let z be the complex number $\frac{1}{2}(1 + i\sqrt{3})$.

Find the modulus and argument of each of the complex numbers $2z^2$ and $\frac{3}{z^2}$.

In an Agrand diagram O represents the origin, A represents the complex number $2z^2$ and B represents the complex number $\frac{3}{z^2}$.

Does the point representing z lie on the line passing through O and B? Justify your answer. The point C is chosen so that OACB is a parallelogram.

Determine, in the cartesian form $p + iq$, the complex number represented by C.

Find the lengths of the diagonals of OABC.

Part B – Dynamics

(3) (a) A particle moves in a straight line with a Simple Harmonic Motion. When the particle is $1m$ and $2m$ from the center of oscillation the velocities are 4 ms^{-1} and 3 ms^{-1} respectively. Find the angular velocity (ω) and the amplitude (a) of the particle.

(b) A particle of mass, 3 kg moves in a straight line. When the displacement of the particle from the center of oscillation (O) is 1 m, velocity is 4 ms^{-1}. At the same time, the force on the particle towards the center O is 12 N. Find the period of oscillation and the amplitude of the particle.

(4) A particle of mass 2 kg is attached to one end of an elastic string of natural length l m whose other end is fixed to a point A on a smooth horizontal plane. The particle is pulled across the plane to a point C where $AC = 1.5\ m$ and is released from rest at C. B is a point on AC such that $AB = 1m$. If the modulus of the string is 10 N, then show that,

(i) From C to B, particle moving with Simple Harmonic Motion at center B.

(ii) The time taken to travel from B to C is $\frac{\sqrt{5}\pi}{10}\ s$.

(iii) The speed of the particle at B is $\frac{\sqrt{5}}{2} ms^{-1}$.

(iv) The particle travels with $\frac{4\sqrt{5}}{5}\ s$ uniform velocity.

Part C - Statics

(5) Show that the center of mass of a uniform arc of circle, radius a subtending an angle 2α at the centre is at a distance $\frac{a \sin \alpha}{\alpha}$ from the center of the circle. Hence, deduce the position of center of gravity of a uniform AB semicircle arc of a radius. Made from the same material thin smooth wire of length 2a is connect with ends of AB. Show that the distance to the center of gravity from the diameter AB of compound object is $\left(\frac{2a}{2+\pi}\right)$. The wire is connected to the point A smoothly and hanging from rest. Show that the slope of the angle of the diameter AB with the vertical axis is $\tan^{-1}\left(\frac{2}{2+\pi}\right)$.

(6) Show that the position of the center of mass of a uniform solid right circular cone of height h on its axis at a distance $\frac{3h}{4}$ from the vertex.

Such a cone, of semi-vertical angle 15^0, rests with its base on a rough horizontal floor. It is tilted to one side by a light inextensible string attached to its vertex. The string pulls downwards making an angle 45^0 with the horizontal, in a vertical plane through the axis of the cone. The edge of the cone is about to slip on the floor, when the vertex is vertically above the point of contact of the edge and the floor. Write down sufficient equations to determine the tension T in the string, The normal reaction and the frictional force. Hence, show that

(i) $T = \frac{3\sqrt{2}\,W}{16}$ (W – weight of the cone)

(ii) the value of the coefficient of friction, is $\frac{3}{9}$.

Part D - Probability

(7) (a) A coin is tossed in three times. Write the universal set for the result.

A=The event {receiving head at first time}, B=The event {receiving the head at second time}, C=The event {receiving head at third time}

(i) Write elements of A, B and C.

(ii) Find, $n(A), n(B), n(C), n(A \cap B), n(B \cap C), n(C \cap A), n(A \cap B \cap C)$.

(b) Prove the De Morgan's law for the following set.

A={1, 2, 3} B={2, 4, 6} U={1, 2, 3, 4, 5, 6, 7, 8}

(c) A and B are two random events such that $P(A \cap B) = \frac{1}{4}$ and

$P(A) = (A/B') = \frac{5}{12}$, Where B' is the complementary event of B. Find,

(i) $P(B/A)$ (ii) $P(B)$ (iii) $P(A/B)$ (iv) $P(A \cup B)$

Are the event A and B mutually elusive? Are they independent? Justify your answer in each case.

(8)(a) A box containing 3 white and 7 black balls. Find the probability of picking out two white balls, if we make out selection without looking.

(b) Let A and B be two events. Define each of the following statements.

(i) Events A and B are independents,

(ii) Events A and B are mutually exclusive,

(iii) Events A and B are exhaustive.

Let the complementary events of A and B be denoted by A' and B' respectively.

Show that $P(A \cap B) + P(A \cap B') = P(A)$

Given that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B') = \frac{1}{2}$, find the value of $P(A' \cap B)$ and the value of $P(A' \cap B')$.

(9) (a) Once of three coins is biased so that the probability of obtaining a head when it is tossed once is p. The other two coins are unbiassed. One of the three coins is chosen at random and tossed twice. Draw a tree-diagram to show the possible outcomes. If the probability of obtaining heads on both tosses is $\frac{17}{54}$, find the value of p.

For this value of p, given that heads were, in fact, obtained on both tosses, find the probability that the chosen coin is biased.

(b) The probabilities of producing nails by machines X, Y and Z are $\frac{3}{5}, \frac{3}{10}$ and $\frac{1}{10}$ respectively. It is known that the $2\%, 3\%$ *and* 4% of nails produced from X, Y and Z are defective.

(i) Find the probability that a nail produced is found to be defective.

(ii) If a nail is found to be defective, Find the probabilities that the nail is produced by X Machine, Y Machine and Z Machine separately.

END.

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Advanced Certificates in Science – Level 2 Part 2

Final Examination – 2020/2021

Duration: Three (03) hours

MHF2522 - Mathematics 4–Paper II

Date: 3rd January 2022 **Time: 1.30 pm –4.30 pm**

Instructions

You are allowed to use non-programmable calculators. Access to mobile phones during the test period is prohibited. A s

Answer five (05) questions including at least two questions from each part.

Part A – Calculus

(1)(a) By making a suitable substitution find the following indefinite integrals.

$$\text{(i)} \int \frac{x^3}{\sqrt{x^2+1}}\,dx \qquad \text{(ii)} \int \sqrt{a^2-x^2}\,dx \qquad \text{(iii)} \int 5^x\,dx$$

(b) By using partial fractions, find the following indefinite integral.

$$\int \frac{x}{(x-1)(x^2+4)}\,dx$$

(2)(a) By using integration by parts, find the following indefinite integral.

$$\int e^x \cos x dx$$

(b) Find the following indefinite integrals.

$$\text{(i)} \int \frac{dx}{\sqrt{6-x-2x^2}} \qquad \text{(ii)} \int \frac{dx}{\sqrt{4x^2-8x-5}}$$

(3)(a) Find the following indefinite integrals.

$$\text{(i)} \int \cos^5 x\, dx \qquad (ii) \int \sin^4 3x \cos^5 3x\, dx$$

(b) Evaluate the following definite integrals.

$$(i) \int_0^1 x \tan^{-1} x\, dx \qquad (ii) \int_1^2 \frac{5x-4}{(1-x+x^2)(2+x)} dx$$

(4)(a) Using the method of integration by parts, show that

$$\int_0^{\frac{\pi}{2}} \sin^6 x\, dx = \frac{5}{6}\int_0^{\frac{\pi}{2}} \sin^4 x\, dx = \frac{5\cdot 3}{6\cdot 4}\int_0^{\frac{\pi}{2}} \sin^2 x dx = \frac{5\pi}{32}$$

(b) Calculate the area enclosed by the curve $y = x(4-x)$, and the x axis.

Part B – Statistics

(5)(a) There four classes with number of students are 12, 20, 18 and 10. The relevant mean of the height of each class are 4.3, 4.6, 4.9 and 5.1. Find the mean height of all the students.

(b) A proof reads through 73 pages manuscript. The number of mistakes found on each of the pages is summarized in the table given below. Determine the mean of the number of mistakes found per page using (i) direct method (ii) assumed mean method.

No. of mistakes	1	2	3	4	5	6	7
No. of pages	5	9	12	17	14	10	6

(6)(a) Draw the cumulative frequency curve for the following data.

Class interval	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89
No. of students	1	6	18	42	68	49	16	4

(b) By using the above data, calculate: Quartiles Q_1, Q_2 and Q_3.

(c) Draw a histogram to represent the frequency distribution and draw frequency polygon on it.

(7) The following are the distribution of ages for 1020 employees in a particular institute.

Age in years	20-24	25-29	30-34	35-39	40-44	45-49	50-54
No. of employees	12	96	198	324	216	107	67

Calculate (i) mean (ii) median (iii) standard deviation (iv) relative measures of dispersion (coefficient of variation) (v) coefficient of skewness

(8) (a) In a frequency distribution, mean = 750, median = 736, mode = 715 and standard deviation = 230 calculate, (i) measure of skewness (ii) coefficient of variation.

(b) The twelve numbers $3, 6, 9, 12, 4, 6, 8, 10, 12, 14, x, y$ have a mode of 6 and a mean of 8. Find

(i) the values of x and y

(ii) the median of the above twelve numbers.

When three additional numbers 8-k, 8, 8+k is included, the variance of the fifteen numbers is found to be 12. Find the values of k.

(c) For a group of 50 male workers the mean and standard deviation of their daily wages are 63 dollars and 9 dollars respectively. For a group of 40 female workers these values are 54 dollars and 6 dollars respectively. Find the mean and variance of the combined group of 90 workers.

END.