The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Mechanical Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: DMX6578 / MEX6278 – Fluid Mechanics**
Academic Year	: 2019/2020
Date	: 30th July 2020
Time	: 09.30-12.30hrs
Duration	: **3 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of 8 questions. All questions carry equal marks.
3. Answer **any 5** questions only.
4. Take acceleration due to gravity and the density of water as **9.81 N/kg** and **1000 kg/m^3** respectively where necessary.

Q1.

a) By considering a differential fluid element, show that the vector form of the continuity equation for a compressible, unsteady fluid flow with usual notation is given by,

$$\frac{d\rho}{dt}+\nabla(\rho u)=0$$

6-marks

Where $\nabla=\left(\frac{\partial}{\partial x}i+\frac{\partial}{\partial y}j+\frac{\partial}{\partial z}k\right)$

b) Obtain the expressions for the continuity equations of following fluid flows.

i. Steady compressible flow

ii Incompressible flow

4-marks

c) The velocity components in x and y direction of an incompressible, steady-flow field are $\mathbf{u=6xt+yz^2}$ and $\mathbf{v=3t+xy^2}$ respectively. Determine the velocity component in the z direction (w), required to satisfy the law of conservation of mass.

10-marks

Q2.

a) Explain the difference between Eularian approach and the Lagrangian approach of analysing the fluid flow. **5-marks**

b) The acceleration **(*a*)** and the angular velocity (*ω*) of a fluid flow, in usual notations, is given by,

Acceleration: $$a = \frac{DV}{Dt} = \frac{\partial V}{\partial t} + u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y} + w\frac{\partial V}{\partial z}$$

and

Angular velocity: $$\omega = \frac{1}{2}\nabla \times V$$

where $V = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$ and $\omega = \omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k}$

15-marks

Find the **acceleration,** the **angular velocity about x-axis,** and the **vorticity vector** at the point **(1, 1, 0)** at **t=2 sec,** of the flow given by the velocity field **V**=x^2z **i** +$4ty^2$ **j** + xzy **k** ms^{-1}.

Q3.

(a). State the Buckingham's *π* theorem. **3-marks**

(b). Show that the frictional torque *τ* of a disc of diameter ***D*** rotating at a speed of ***N*** in a fluid of dynamic viscosity *μ* and the density *ρ* in a turbulent flow is given by, **7-marks**

$$\frac{\tau}{D^5N^2\rho} = \emptyset\left(\frac{\mu}{D^2N\rho}\right)$$

(c). In order to predict the torque on a disc 0.5 m diameter which rotates in oil at 200 rev/min, a model is made to a scale of 1/5. The model is rotated in water. Calculate the speed of rotation for the model which produces dynamic similarity. **5-marks**

(d). When the model is tested at 18.75 rev/min the torque was 0.02 Nm. Predict the torque on the full- size disc at 200 rev/min. **5-marks**

	Density (kg/m^3)	Dynamic viscosity (Ns/m^2)
Oil	750	0.2
Water	1000	0.001

Q4.

Equation of a fluid in rigid body motion is given by $\nabla P + (\rho.g).k = -\rho a$

Where $a = a_x i + a_y j + a_z k$

(a). For fluids at rest or moving on a straight path (parallel to x or y directions) at a constant velocity, show that the pressure gradients along the x, y and z directions are given by;

5-marks

$$\frac{\partial p}{\partial x} = 0 \quad , \quad \frac{\partial p}{\partial y} = 0 \quad \text{and} \quad \frac{\partial p}{\partial z} = -\rho g$$

(b). An oil tanker of rectangular section of height 2.5m and length 4m contains oil of density 900 kg/m^3 to depth 2m (Figure Q4-a). If the tanker is moving with uniform acceleration a_x in the x direction (Figure Q4-b), show that the slop of the constant pressure surface is given by,

$$\tan(\theta) = \frac{a_x}{g}$$

8-marks

(c). Determine the maximum uniform acceleration for no spillage of oil.

7-marks

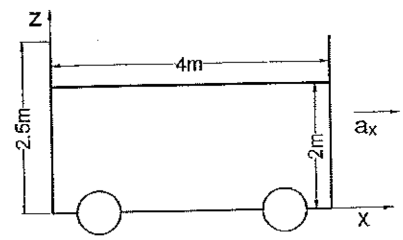

Figure Q4-a

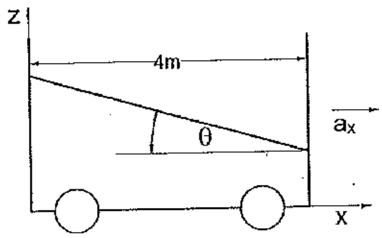

Figure Q4-b

Q5.

(a). Explain the significance of the boundary layer displacement thickness **(δ^*)** and the momentum thickness **(Θ),** and express them as integrals of the boundary layer velocity profiles for a smooth flat plate.

5-marks

(b). A laminar boundry layer is given by the following velocity profile.

$$\frac{u}{u_\infty} = \alpha\left(\frac{y}{\delta}\right) + \beta\left(\frac{y}{\delta}\right)^3$$

10-marks

Where α and β are arbitrary constants

i. Find α and β by using the boundary conditions

ii. Show that **(δ^*/δ) =3/8**

(c). Calculate the frictional drag force per unit width **(Fd)** due to the boundary.

5-marks

Q6.

(a). By considering the forces acting on the differential control volume of a fluid, write down the expression for the **surface forces** and the **body forces**. 5-marks

(b). A thin layer of a incompressible liquid flows down in an inclined plane steadily, as shown in **Figure Q6**. Starting from Navier-Stokes equations, show that for steady flow the momentum equation in x-direction can be given expressed as,

$$\rho g \sin\theta + \mu \frac{\partial^2 u}{\partial y^2} = 0$$ 5-marks

(b). If h is the thickness of the layer in the fully developed stage, show that the velocity distribution is given by,

$$u(y) = \frac{-\rho g \sin\theta}{2\mu}\left(y^2 - h^2\right)$$ 5-marks

(c). Show that the volume flow rate per unit width is given by,

$$Q = \left(\frac{\rho g h^3}{3\mu}\right)\sin\theta$$ 5-marks

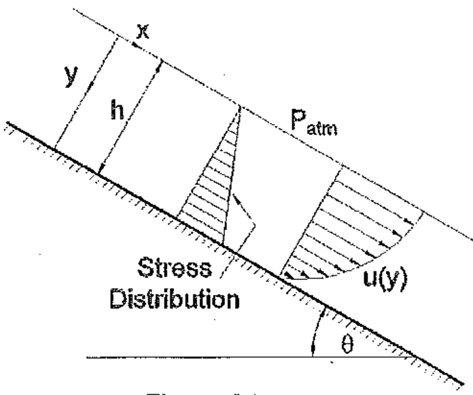

Figure Q6

Q7.

(a). Describe the following terms, 6-marks

I. Doublet
II. Rankine Oval
III. Singularity

10-marks

(b). A source with strength **0.25m²s⁻¹** and a vortex with strength **1m²s⁻¹** (counter clockwise) are located at the origin point. Determine the equation for stream function and velocity potential.

(c). Find out the velocity components at the point ***P(1,0.5)*** 4-marks

Q8.

(a). State the applications of potential flow theory. **5-marks**

(b). In a two-dimensional incompressible flow field the velocity components for the x and the y directions are

$u = x - 4y$ and $v = -y - 4x$ respectively. **8-marks**

i. Show that the velocity potential exists, and determine the function.

ii. Find out the stream function and show that, it satisfies the Laplace equation. **7-marks**

Nervier-Stokes equations for incompressible flow:-

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\rho g_x-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\rho g_y-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=\rho g_z-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

Frictional drag force per unit width due to laminar boundary layer :- $F_d = \rho\int_0^{\delta} u(U_\infty - u)dy$

Acceleration vector:- $\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$

Cartesian Coordinates:- $\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$

Cylindrical Coordinates:- $\nabla = \frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial}{\partial \theta} + \frac{\partial}{\partial z}$

------------------------ ***End of Paper*** ------------------------