The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Electrical and Computer Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**:EEX6534 /ECX6234**
Academic Year	: 2019/20
Date	: 31st July 2020
Time	: 0930-1230hrs
Duration	: **3 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Eight (8)** questions in **Four (4)** pages.
3. Answer any **Five (5)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Important formulas are provided.
6. This is a Close Book Test **(CBT).**
7. Answers should be in clear hand writing.
8. Do not use red colour pen.

1.

(a) Impulse train $p(t)$ is given by $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$

(i) Express $p(t)$ as an exponential Fourier series. (Calculation of Fourier coefficients (C_n) is not Required). [3 *Marks*]

(ii) If $p(t)$ is used to sample a band limited signal $x(t)$, write an expression for the sampled signal using the answer to Q 1 (a) (i). [3 *Marks*]

(iii) Show that the frequency spectrum of the sampled signal is given by $\sum_{n=-\infty}^{\infty} X(\omega - n\omega_0)$, where $\omega_0 = 2\pi \frac{1}{T}$. Take $C_n = 1$. [6 *Marks*]

(iv) If the bandwidth of $x(t)$ is B, sketch the frequency spectrum of the sampled signal.

[4 *Marks*]

(b) How would you discretize an analog signal so that no information of the original signal is lost?

[4 *Marks*]

2.

(a) A discrete rectangular pulse $x[n]$ is shown in Fig.2(a).

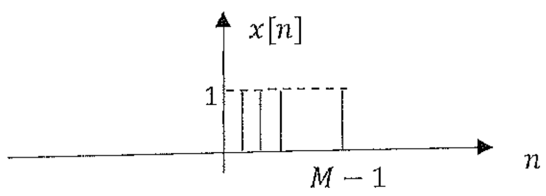

Fig. 2 (a)

(i) Find the Discrete Time Fourier Transform ($DTFT$) $X(\omega)$ of the pulse. [4 *Marks*]

(ii) Sketch $X(\omega)$ indicating the important values. [2 *Marks*]

(b) The discrete signal $x[n]$ is transformed into it's analog form using a Zeroth Order Hold (ZOH) reconstructor. A ZOH reconstructor uses the interpolating function $g(t)$ as shown in Fig. 2 (b). The signal $x[n] = \cos[\omega_0 n]$ is applied as the input to the reconstrucor. The output of the reconstructor is the analog signal $x(t)$.

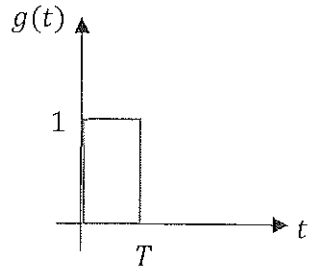

Fig. 2 (b)

(i) Write the relationship between $x[n]$, $g(t)$ and $x(t)$. [4 *Marks*]

(ii) Find a suitable value for T if the sampling frequency is f_s. [2 *Marks*]

(iii) Sketch the output of the reconstructor. [3 *Marks*]

(iv) Show that $X(F) = G(F).X(\omega)|_{\omega=\frac{2\pi F}{F_s}}$, where $X(F)$ and $G(F)$ are the Fourier transforms of $x(t)$ and $g(t)$ respectively. $X(\omega)$ is the $DTFT$ of $x[n]$. [5 *Marks*]

3.

(a) An LTI system is represented by the difference equation $y[n] = x[n] + 2x[n-1] + x[n-2]$, where $x[n]$ and $y[n]$ are the input and the output to the system respectively.

(i) Show that the equation represents a lowpass filter. [6 *Marks*]

(ii) Find the 3-dB bandwidth of the filter. [2 *Marks*]

(iii) If the impulse response of the filter is $h[n]$, sketch the frequncy response of the LTI system whose impulse response is $(-1)^n\, h[n]$. [4 *Marks*]

(iv) What kind of filter is represented by the system mentioned in (iii)? Justify your answer. [2 *Marks*]

(b) (i) Write an expression for the Discrete Fourier Transform (DFT) of a signal $x[n]$. [3 *Marks*]

(ii) Explain why DFT is preferred over $DTFT$ in the estimation of spectral components of a signal in practical situations. [3 *Marks*]

4.

(a) A discrete rectangular pulse $x[n]$ has L discrete samples (0 .....$L-1$). The pulse height is 1. Now $(N-L)$ zero's are added to $x[n]$ to form a new pulse $x_N[n]$.

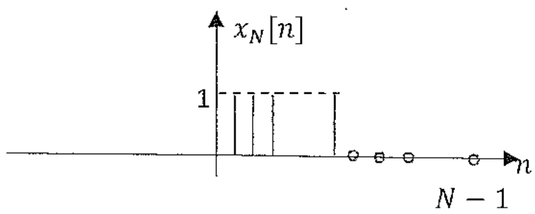

Fig. 4 (a)

(i) Find the Discrete Fourier Transform (DFT) $X_N(k)$ of $x_N[n]$. [6 *Marks*]

(ii) Sketch $|X_N(k)|$ and find the total number of spectral lines in the main lobe. [3 *Marks*]

(iii) Based on the results of 4 (a) (i) and 4 (a) (ii), explain how the number of zeros added influences the frequency spectrum of the signal. [4 *Marks*]

(b) If no zeros are added to the original signal

(i) find the DFT of the original signal and sketch the magnitude spectrum $|X(k)|$.

[3 *Marks*]

(ii) explain the result in 4(b) (i). [4 *Marks*]

5.

The DFT of a sequence $x[n], n = 0,1, \dots, N-1$ can be written as $X_N[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn}$,

$k = 0, \dots N-1$, where $W_N = e^{-j\frac{2\pi}{N}}$. We can rearrange the terms of the above sequence as

$$X_N[k] = \sum_{m=0}^{\frac{N}{2}-1} x[2m] W_N^{2mk} + \sum_{m=0}^{\frac{N}{2}-1} x[2m+1] W_N^{(2m+1)k}$$

Here $0^{th}, 2^{nd}, 4^{th}, \dots$ terms and $1^{st}, 3^{rd}, 5^{th}$ ..... terms are seperately considered.

(a) Show that $W_N^2 = W_{N/2}$. [5 *Marks*]

(b) Using the result of (a) show that $X_N[k] = X_{N/2}^{even}[k] + W_N^k X_{N/2}^{odd}[k]$. [9 *Marks*]

$X_{N/2}^{even}[k]$ is the DFT of $0^{th}, 2^{nd}, 4^{th}, \dots$ terms and

$X_{N/2}^{odd}[k]$ is the DFT of $1^{st}, 3^{rd}, 5^{th}$ ..... terms.

Assume that N is an even number.

(c) Applying the same procedure to $X_{N/2}^{even}[k]$ and $X_{N/2}^{odd}[k]$, they can be simplified into smaller sequences and hence calculation of DFT's can be further simplified.

What should be the format of N so that calculation of DFT's is possible using minimal number of terms? [6 *Marks*]

6.

(a) Sketch the frequency response of a digital lowpass filter with cutoff frequency ω_c if

(i) the filter is ideal. [3 *Marks*]

(ii) the filter is non-ideal. Also indicate all the important values and important regions.

[3 *Marks*]

(b) A digital lowpass filter has a passband frequency of 5 kHz with a 1 dB ripple. Its stopband frequency and the stopband attenuation are 5.5 kHz and 40 dB respectively. The sampling frequency of the filter is 22 kHz.

(i) Express passband- and stopband frequencies in radians. [3 *Marks*]

(ii) Sketch the magnitude response of the filter indicating all the important values for the frequency range $0 < \omega < \pi$. [7 *Marks*]

(c) Using $Paley - Wiener$ theorem show that the filter in (a)(i) is non-realizable. [4 *Marks*]

7.

(a) An analog filter has a transfer function $H(s) = \frac{1}{S+2}$

(i) Using Euler approximation design a digital filter to approximate the above filter. Give the transfer function $H_D(z)$ of the filter. Use the sampling frequency of 6 kHz .

[5 *Marks*]

(ii) Find the impulse response of the filter. [5 *Marks*]

(b) A digital lowpass filter is to be designed to satisfy the following characteristics:

Passband frequency: 5 kHz; Stopband frequency: 6 kHz; Sampling frequency: $F_s = 30\ kHz$. A window function with transition band $\Delta\omega = \frac{8\pi}{N}$ is used in the design of the filter, where N is the window size.

(i) Decide a suitable value for N. [5 *Marks*]

(ii) How would you achieve the causality of the filter? [5 *Marks*]

8.

(a) Briefly explain how a Kalman filter functions using a block diagram. [9 *Marks*]

(b) Briefly explain the following:

(i) $ARMA$ process. [4 *Marks*]

(ii) Blind deconvolution. [4 *Marks*]

(iii) Give a practical application of blind deconvolution. [3 *Marks*]

<u>Some useful Transforms and equations</u>

Fourier transform ($X(F)$) of $x(t)$: $X(F) = \int_{-\infty}^{\infty} x(t).\,e^{-j2\pi Ft}\,dt$

Discrete Time Fourier Transform of $x[n]$: $X(\omega) = \sum_{-\infty}^{\infty} x[n]e^{-j\omega n}$

z –Transform of $x[n]$: $X(z) = \sum_{-\infty}^{\infty} x[n]z^{-n}$

Discrete Fourier Transform of $x[n]$: $X(k) = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}nk}$; $k = 0,1, \ldots\ldots N-1$

Pass-band ripple of a lowpass filter expressed in dB = $20\ log\left(\frac{1}{1-\delta}\right)$ dB

Stop-band attenuation of a lowpass filter expressed in dB = $20\ log(\delta_s)$ dB