00158

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Electrical and Computer Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: EEX6541/ECX6241 Field Theory**
Academic Year	: 2019/20
Date	: 13th August 2020
Time	: 0930 - 1230 hrs.
Duration	: 03 hours

General Instructions

1. Read all instructions carefully before answering the questions
2. This is a Closed Book Test (CBT).
3. This question paper consists of **Six (06)** questions in **Three (03)** pages.
4. Answer **only five (05) questions** by answering **ALL in Section A** and selecting **two (02) from Section B**.
5. All questions carry equal marks.
6. Answer for each question should commence from a new page.
7. Answers should be in clear handwriting.
8. Do not use Red color pen.
9. Assume any missing parameters with suitable values

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Section A

Answer all questions in this section.

Q1.

a. Given that $A = \frac{x^3}{3}\boldsymbol{a}_x$, evaluate both sides of the divergence theorem for the volume of a cube with 1 m on an edge, centered at the origin and the edges parallel to the axes.

[10]

b. Consider a uniformly charged ring of radius R and charge density λ. What is the electric potential at a point of distance z away from the central axis?

[10]

Q2.

a. State and explain the Coulomb's law in electrostatics. Express it mathematically using two point charges. [05]

b. The permittivity of the dielectric material between the plates of a parallel-plate capacitor varies uniformly from ε_1 in one plate to ε_2 in the other plate. Show that the capacitance is given by

$$C = \frac{A}{d}\frac{\varepsilon_2 - \varepsilon_1}{\ln\left(\varepsilon_2/\varepsilon_1\right)}$$

where A and d denote the area of plates and the separation between plates, respectively. [12]

c. Find the value of C for $\varepsilon_1 = \varepsilon_2$. [03]

Q3.

a. The current density in a certain region is given by

$$\boldsymbol{J} = \frac{5}{r}\boldsymbol{a}_r + \frac{10}{(r^2+1)}\boldsymbol{a}_z \qquad A/m^2$$

Determine the total current crossing the surface at $z = 3$ and $r < 6$ in the z-direction.

[10]

b. Two coaxial circular wires of radii a and b ($b > a$) are separated by a distance h such that $h \gg a, b$. Determine the mutual inductance between the wires.

[10]

Section B

Select **only two** questions from this section.

Q4.

a. Starting from Maxwell's equations $\nabla \times \boldsymbol{E} = -\frac{\partial B}{\partial t}$ and $\nabla \times \boldsymbol{H} = J + \frac{\partial D}{\partial t}$, show that $\nabla \cdot \boldsymbol{B} = 0$ and $\nabla \cdot \boldsymbol{D} = \rho$.

[06]

b. The electric and magnetic fields in the free space are given by

$$\boldsymbol{E} = \frac{50}{\rho}\cos(10^6 t + \beta z)\boldsymbol{a}_\phi$$
$$\boldsymbol{H} = \frac{H_0}{\rho}\cos(10^6 t + \beta z)\boldsymbol{a}_\rho$$

Express the above fields in phasor form and determine the constants H_0 and β such that fields satisfy Maxwell's equations.

[14]

Q5.

a. Define the Poynting Vector and state the Poynting Theorem.

[06]

b. The electric field of a uniform plane wave propagating in the positive z-direction is given by

$$\boldsymbol{E} = E_0\cos(\omega t - \beta z)\,\boldsymbol{a}_x + E_0\sin(\omega t - \beta z)\,\boldsymbol{a}_y$$

where E_0 is a constant. Determine

i. the corresponding magnetic field $\boldsymbol{H}$.
ii. the Poynting vector.

[14]

Q6. Briefly explain the following terms.

a. Permittivity
b. Permeability
c. Self-inductance
d. Mutual inductance
e. Skin effect

[04 × 5]

-end-

Note:

Cylindrical Coordinates

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla \cdot \boldsymbol{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \boldsymbol{A} = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)\hat{\boldsymbol{\rho}} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\hat{\boldsymbol{\varphi}} + \frac{1}{\rho}\left(\frac{\partial}{\partial \rho}(\rho A_\varphi) - \frac{\partial A_\rho}{\partial \varphi}\right)\hat{\mathbf{z}}$$

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

Spherical Coordinates

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}},$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta A_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial \varphi},$$

$$\begin{aligned}\nabla \times \mathbf{A} = {} & \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial \theta}\left(A_\varphi \sin\theta\right) - \frac{\partial A_\theta}{\partial \varphi}\right)\hat{\mathbf{r}} \\ & + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{\partial}{\partial r}\left(r A_\varphi\right)\right)\hat{\boldsymbol{\theta}} \\ & + \frac{1}{r}\left(\frac{\partial}{\partial r}\left(r A_\theta\right) - \frac{\partial A_r}{\partial \theta}\right)\hat{\boldsymbol{\varphi}},\end{aligned}$$

$$\begin{aligned}\nabla^2 f &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2} \\ &= \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right)f + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right)f + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial \varphi^2}f.\end{aligned}$$