The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Civil Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: CVX 5531 Mechanics of Fluids/CEX 5231 Mechanics of Fluids**
Academic Year	: 2019/20
Date	: 02nd August 2020
Time	: 0930-1230hrs
Duration	**: 03 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **SEVEN (07)** questions on **FIVE (05)** pages.
3. Answer any **FIVE (05)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Necessary additional information is provided.
6. This is a Closed Book Test (CBT).
7. Answers should be in clear hand writing.
8. Do not use Red colour pen.
9. Take,

   Density of water = 1000 kgm^{-3} Acceleration due to gravity = 9.81 ms^{-2}

   Kinematic viscosity of water = 1.12 x 10^{-06} m^2/s

Question 01

(a) For an incompressible flow, the fluid velocity components in three dimensional space are described by:

$u = a(x^2 - y^2)$ $\qquad v = -2axy$ $\qquad w = 0$

Show that the flow satisfies continuity equation.

(03 marks)

(b) Show that the flow field in (a) is an exact solution to the Navier-Stokes equations by considering, $g_x = g_y = 0$ and $g_z = -g$.

(09 marks)

(c) Obtain the corresponding pressure distribution.

(08 marks)

Additional Information:

Navier-Stokes equations for unsteady flow in three-dimensional Cartesian coordinate system are given by,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + g_x$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + g_y$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) + g_z$$

Question 02

(a) Briefly explain what is meant by dynamic similarity in model studies.

(04 marks)

(b) A fluid of kinematic viscosity, ν, density, ρ and surface tension, σ flows over a rectangular weir of width, B.

(i) Using Buckingham's Pi theorem, show that a rational formula for the flow over a rectangular weir, Q is given by,

$$Q = g^{1/2}h^{5/2}\phi\left(\frac{g^{1/2}h^{3/2}}{\nu}, \frac{gh^2\rho}{\tau}, \frac{B}{h}\right)$$

where h is the head above the sill, and g is the acceleration due to gravity.

(ii) Experiments with water on a rectangular weir, demonstrates that a practical formula for flow is given by $Q = 1.84Bh^{1.47}$, Q and h being in SI units. Neglecting surface tension, estimate the percentage error involved, if this formula is used for measuring the flow of oil whose kinematic viscosity is eight (08) times that of water.

(16 marks)

Question 03

(a) The shear stress on the solid surface of a turbulent boundary layer is given by,

$$\tau_0 = 0.0225\rho U_s^2 \left(\frac{\nu}{U_s\delta}\right)^{1/4}$$

where, ρ and ν are the density and the kinematic viscosity, respectively of the flowing fluid, U_s is the free stream velocity and δ is the boundary layer thickness. The velocity distribution within the boundary layer is represented by the power law, $\frac{u}{U_s} = \left(\frac{y}{\delta}\right)^{1/7}$, where u is the velocity at a distance y from solid surface.

(i) Show that the boundary layer thickness, $\delta = 0.37\left(\frac{\nu}{U_s}\right)^{1/5} x^{4/5}$

(ii) If the boundary layer thickness is 3.2 mm at a given section, determine the corresponding momentum thickness

(10 marks)

(b) A passenger train is supposed to move through air of density 1.22 kg/m^3 and dynamic viscosity of 1.79 x 10^{-05} Ns/m^2. The train is 120.0 m long, 2.8 m wide and 2.75 m high. If the maximum expected speed of the train is 150 km/hr, calculate the maximum power required to overcome the skin friction. You may assume that a turbulent boundary layer prevails from the leading edge of the train.

(10 marks)

Additional Information:

Momentum thickness of a boundary layer is given by: $\theta = \int_0^\delta \frac{u}{U_s}\left(1 - \frac{u}{U_s}\right) dy$

Shear stress on the solid surface of a boundary layer is given by: $\tau_0 = \rho U_s^2 \frac{d\theta}{dx}$

Question 04

(a) A concrete channel of triangular cross-section is to be constructed for irrigation purposes.

(i) Show that the hydraulic radius, R for the most economical triangular cross-section is given by, $R = \sqrt{2}y/4$, where y is the depth of flow in the channel.

(ii) Prove that for a channel with a triangular cross section, the critical depth, $y_c = 4H/5$, where H is the specific energy.

(08 marks)

(b) A hydraulic jump occurs in a horizontal 90^0 triangular channel. If the sequent depths in this channel are 0.55m and 1.10 m respectively, estimate;

(i) The flow rate along the channel and.

(ii) The energy loss in the jump.

(12 marks)

Question 05

(a) Considering the compression of fluid and expansion of pipe material, show that the expression for pressure rise in a supply tunnel (which is restrained at the ends) of a hydropower scheme following a sudden complete valve closure is given by,

$$\delta p = v_0 \sqrt{\frac{\rho}{\frac{1}{K} + \frac{(1-\nu^2)d}{Et}}}$$

where v_0 is the initial velocity of flow in the supply tunnel, ρ and K are the density and the bulk modulus of the fluid, d and t are the diameter and the thickness of the pipe, and E and ν are the modulus of elasticity and Poisson's ratio of pipe material, respectively.

(10 marks)

(b) A steel pipeline is 300 mm in diameter and has a wall thickness of 5 mm. The pipe is 1000 m long and conveys water at a flow rate of 0.1 m^3/s. The static pressure at the outlet is 1200 kN/m^2. Disregarding any friction effect,

(i) Calculate the total pressure in the pipe due a sudden complete valve closure.

(ii) Calculate the critical time of closure.

(iii) Plot the temporal variation of water hammer pressure at mid-point of the pipe

(10 marks)

The bulk modulus of water, $K = 2.20 \times 10^{06}$ kN/m^2

The modulus of elasticity of steel, $E = 2.10 \times 10^{08}$ kN/m^2

The Poisson's ratio of steel, $\nu = 0.41$

Additional Information:

The velocity of pressure wave, $C = \frac{\delta p}{\rho v_0}$

The critical time of valve closure, $T_0 = 2L/C$

Question 06

(a) Show that an expression for the specific speed, N_s of a turbine in terms of its speed, N, output power, P, and head, H is given by,

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

(06 marks)

(b) A double jet Pelton wheel turbine has a specific speed of 15 and is required to deliver 1120kW. The turbine is supplied through a pipeline from a reservoir whose level is 395 m above the nozzles. Allowing 5% for friction loss in the pipe, calculate the,

(i) Speed of the wheel in rpm

(ii) Mean diameter of bucket circle

(iii) Diameter of jets

The coefficient of velocity at the nozzle is 0.98, the speed ratio is 0.46 and the overall efficiency of the turbine is 86%.

(14 marks)

Additional Information:

The specific speed of a turbine is the speed of a geometrically similar turbine which produces 1 kW of power under a head of 1 m.

Question 07

(a) Referring to Figure Q6, show that the Euler's equation for the theoretical head developed by a centrifugal pump is given by,

$$H_{th} = \frac{v_{w2}u_2}{g}$$

(06 marks)

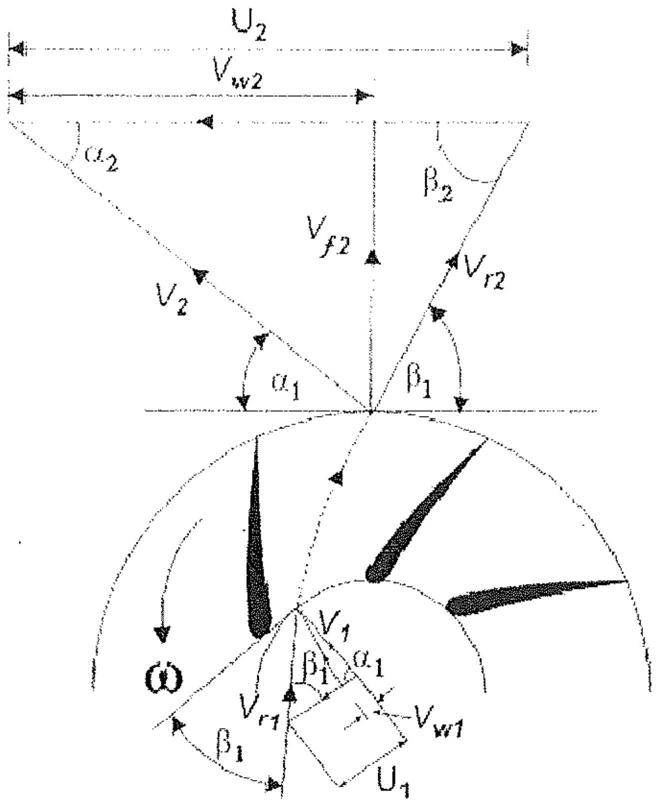

Figure Q6

(b) A centrifugal pump running at 1120 rev/min gave the following relation between head and discharge with the best efficiency point observed at a discharge of 19.0 m^3/min.

Discharge (m^3/min)	0	6.0	12.0	18.0	24.0	30.0
Head (m)	50.0	49.5	48.0	42.0	28.0	0

A pump with above characteristics is used to convey water from one reservoir to another, which is 24.0 m higher in elevation while running the impeller at a speed of 1120 rev/min. The pipeline connecting the two reservoirs is 315 m long and has a diameter of 0.3 m. Friction factor, f for the pipe is found to be 0.005.

(i) Estimate the operating head and corresponding discharge

(ii) To what speed should the pump be changed to operate at the best efficiency point?

(14 marks)

Friction head loss, $h_f = 4f\frac{l}{d}\frac{v^2}{2g}$