00168

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Civil Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: CEX5233/CVX5533 Structural Analysis**
Academic Year	: 2019/20
Date	: 24 July 2020
Time	: 0930-1230hrs
Duration	**: 3 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Eight (8)** questions in **Five (5)** pages.
3. Answer any **<u>Five (5)</u>** questions only. All questions carry equal marks.
5. Answer for each question should commence from a new page.
6. This is Closed Book Test (CBT).
7. Answers should be in clear hand writing.
8. Do not use Red colour pen.

QUESTION 1

(i) Generalized Hooke's law with usual notation is given as

$$\sigma_{ij} = C_{ijpq}\epsilon_{pq}$$

where C_{ijpq} is the elastic constant matrix. Show that there are 21 elastic constants for a general three dimensional stress field. (2 Marks)

(ii) Strain tensor for a homogenous, isotropic material can be written with usual notation as

$$\epsilon_{ij} = \frac{\sigma_{ij}}{2\mu} - \frac{\lambda}{2\mu(3\lambda + 2\mu)}\sigma_{pp}\delta_{ij}$$

where λ and μ are Lame constants. δ_{ij} is the Kronecker delta. Using above expression obtain six independent strain components. (3 Marks)

(iii) A cube of side **"a"** made from a material having Young's modulus **"E"** and Poisson's ratio **"υ"** is kept inside a rigid block and pressed by applying a pressure **P** on top face as shown in Figure **Q1**. Assuming material is in a uniform stress state, find

(a) All six stress components. (6 Marks)

(b) All six strain components. (6 Marks)

(c) Dimensions of the deformed cube. (3 Marks)

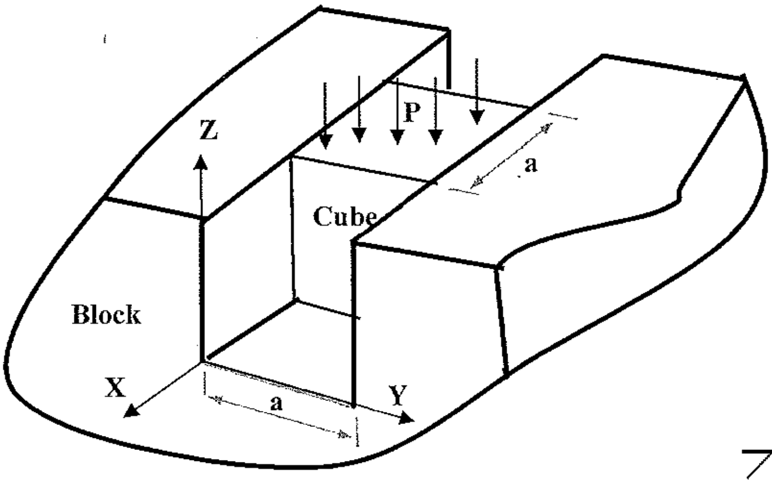


Figure Q1

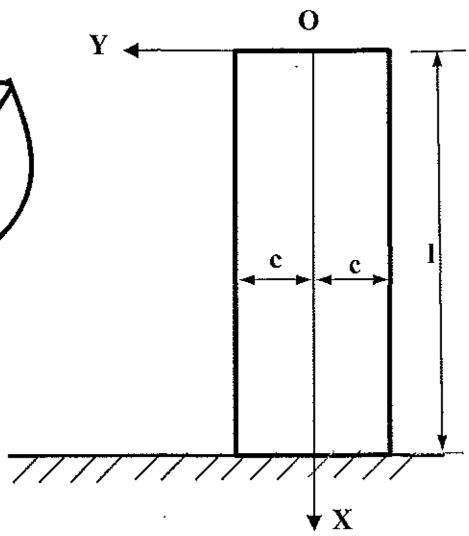


Figure Q2

QUESTION 2

(i) Airy's stress function is a mathematical function used in structural analysis. List two conditions for the application of Airy's stress function. (2 Marks)

(ii) A concrete gravity dam of rectangular cross section is shown in Figure **Q2**. It is subjected to water pressure load (Specific weight of water is **q**), and its self-weight (Concrete specific weight is $\mathbf{q_1}$). It is in a state of plane strain and following stress field was found using a mathematical function.

$$\sigma_{xx} = \frac{qx^3y}{4c^3} + \frac{q}{4c^3}\left(-2xy^3 + \frac{6}{5}c^2xy\right) - q_1x$$

$$\sigma_{yy} = -\frac{qx}{2} + qx\left(\frac{y^3}{4c^3} - \frac{3y}{4c}\right)$$

$$\tau_{xy} = \frac{3qx^2}{8c^3}(c^2 - y^2) - \frac{q}{8c^3}(c^4 - y^4) + \frac{q}{4c^3}\frac{3}{5}c^2(c^2 - y^2)$$

(a) Write down the equations of equilibrium applicable to this problem and check whether this stress field satisfied equilibrium. (6 Marks)

(b) Give Stress boundary conditions for this problem and check whether this stress field satisfies these boundary conditions. (6 Marks)

(c) Plot the distribution of stress components, taking c = 3.0 m, l = 10.0 m, q = 10 kN/m^3, and q_1 = 24 kN/m^3. (6 Marks)

QUESTION 3

(i) Briefly explain what you understand by **Compatibility Conditions**" in continuum mechanics. (3 Marks)

(ii) Given state of strains at a point with respect to a convenient coordinate system (X, Y and Z) be $\varepsilon_{XX} = -3000\mu\epsilon$, $\varepsilon_{YY} = 2000\mu\epsilon$, $\varepsilon_{ZZ} = -2000\mu\epsilon$, $\gamma_{XY} = -5830\mu\epsilon$, $\gamma_{YZ} = -670\mu\epsilon$, $\gamma_{XZ} = -3000\mu\epsilon$

a) Write the strain tensor in matrix form. (4 Marks)

b) Determine the strain invariants (***J₁, J₂ & J₃***). (6 Marks)

c) Show that principal strains are 0.00350, -0.00162, -0.00488. (4 Marks)

d) Determine the maximum shear strain. (3 Marks)

QUESTION 4

(i) Explain why statically indeterminate structures are preferred over statically determinate structures. (4 Marks)

(ii) A continuous beam (ABCD) is shown in Figure Q4. Flexural rigidities of members AB and CD are equal to EI and member BC is 2EI. Uniformly distributed load (w) is acting on members, AB and BC and a concentrated load wl in member CD, respectively.

a) Determine the degree of statical indeterminacy of the beam. (3 Marks)

b) Draw a released structure. (3 Marks)

c) Determine the flexibility matrix for the drawn released structure. (4 Marks)

d) Determine bending moments at B and C. (6 Marks)

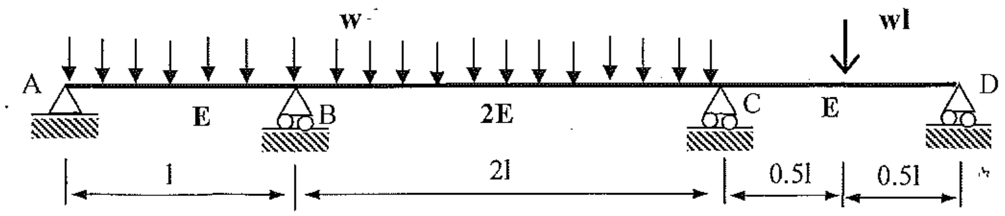


Figure Q4

QUESTION 5

(i) Explain how "**Kinematic Indeterminacy**" of a structure varies from **degree of statical indeterminacy** (4 Marks)

(ii) A portal frame structure shown in Figure Q5. Flexural rigidities of members are same. Find the free nodal displacements at B using the displacement method. You can neglect the axial deformation. (10 Marks)

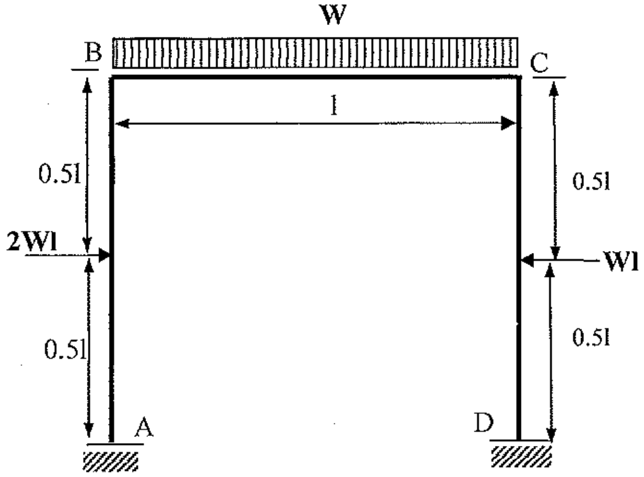


Figure Q5

(iii) Using above results, determine the bending moment at B. (6 Marks)

QUESTION 6

(i) List three assumptions used in the analysis of thin plates with small deflections. (4 Marks)

(ii) Write strain-displacement relations for a rectangular plate with usual notations. (6 marks)

(iii) Describe how strain gauge rosette is used in determining the stress state of a point.

Note: You can use Mohr's circle for strains to explain your problem. (4 marks)

(iv) The stress state of a certain bracket was determined using a strain rosette as shown in Figure Q6. Due to the loadings, strain gauges gave strain values as $\epsilon_a = 60\ \mu\epsilon$, $\epsilon_b = 120\ \mu\epsilon$, $\epsilon_c = 30\ \mu\epsilon$. Determine the in-plane principal strains. (6 marks)

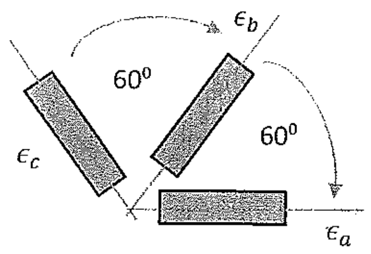


Figure Q6

QUESTION 7

(i) Describe conceptual difference between "**lower bound solution**" and "**upper bound solution**" in plastic design of beams and frames. (2 Marks)

(ii) A two-bay frame structure is shown in Figure Q7. Dimensions and plastic moments of the columns and beam are given in the figure.

(a) Draw possible locations of plastic hinge formations. (2 Marks)

(b) Draw elementary failure mechanisms. (2 Marks)

(c) Determine load factors for each elementary failure mechanism. (6 Marks)

(d) Determine the most probable failure mechanism by combining elementary failure mechanisms. (6 Marks)

(e) Explain how you can ensure the unique solution. (2 Marks)

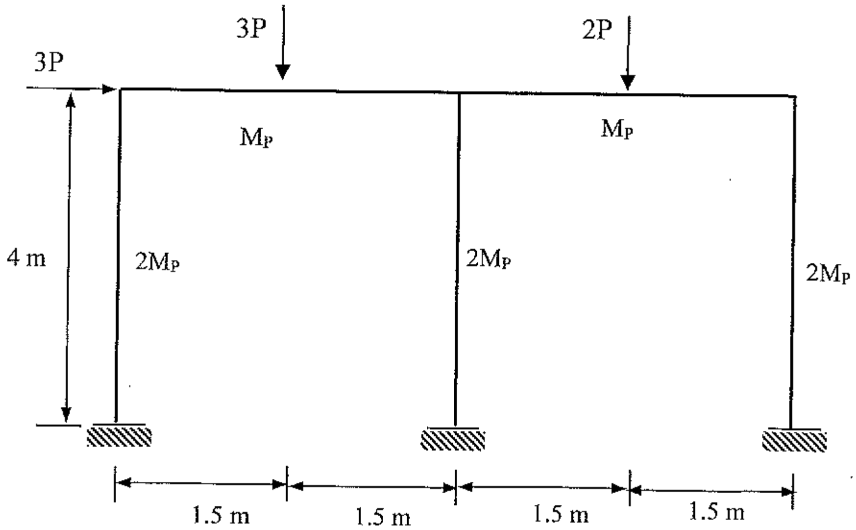


Figure Q7

QUESTION 8

(i) List two assumptions used in membrane theory of thin shells. (3 Marks)

(ii) A cylindrical tank of radius "**a**" and thickness "**h**" is covered with a circular slab plate. Circular plate is subjected to uniformly distributed load **q/unit area** as shown in Figure Q8. The rotation stiffness at the joint is k.

(a) Obtain an expression for radial moment (M_r) of the plate. (8 Marks)

(b) Determine the maximum radial moment of the plate. (4 Marks)

Note: Governing equation for axi symmetric circular plates

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = \frac{qr}{2D} + \frac{C_1}{r}$$

And radial moment is $M_r = -D\left[\frac{d^2W}{dr^2} + \frac{v}{r}\frac{dw}{dr}\right]$ with standard notations.

(iii) Briefly explain how energy methods are used in analysis of plate problems in obtaining approximate solutions. (5 Marks)

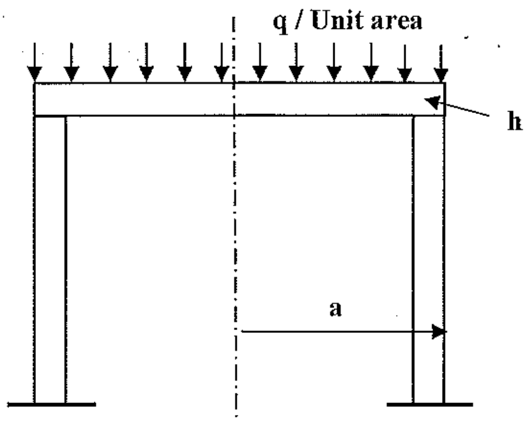
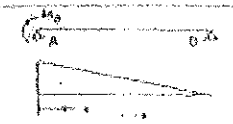
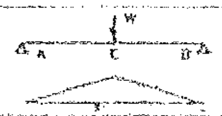
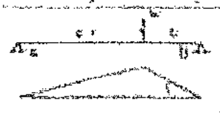
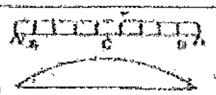
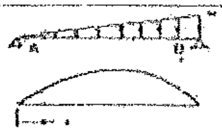
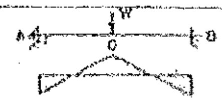
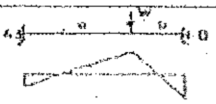
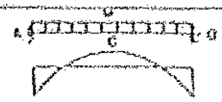
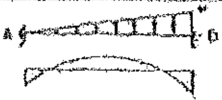


Figure Q8

Table 1 Formulas for Beams

Structure	Shear	Moment	Slope	Deflection
Simply supported Beam				
	$S_A = -\frac{M_o}{L}$	M_o	$\theta_A = \frac{M_o L}{3EI}$ $\theta_B = -\frac{M_o L}{6EI}$	$Y_{max} = 0.062\frac{M_o L^2}{EI}$ $at\ x = 0.422L$
	$S_A = \frac{W}{2}$	$M_c = \frac{WL}{4}$	$\theta_A = -\theta_B = \frac{WL^2}{16EI}$	$Y_c = \frac{WL^3}{48EI}$
	$S_A = \frac{Wb}{L}$ $S_B = \frac{Wa}{L}$	$M_D = \frac{Wab}{L}$	$\theta_A = \frac{Wab}{6EIL}(L+b)$ $\theta_B = -\frac{Wab}{6EIL}(L+a)$	$Y_D = \frac{Wa^2b^2}{3EIL}$
	$S_A = \frac{WL}{2}$	$M_c = \frac{WL^2}{8}$	$\theta_A = -\theta_B = \frac{WL^3}{24EI}$	$Y_c = \frac{5WL^4}{384EI}$
	$S_A = \frac{WL}{6}$ $S_B = \frac{WL}{3}$	$M_{max} = 0.064WL^2$ $at\ x = 0.577L$	$\theta_A = \frac{7WL^3}{360EI}$ $\theta_B = -\frac{8WL^3}{360EI}$	$Y_{max} = 0.00652\frac{WL^4}{EI}$ $at\ x = 0.519L$
	$S_A = \frac{WL}{4}$	$M_c = \frac{WL^2}{12}$	$\theta_A = -\theta_B = \frac{5WL^3}{192EI}$	$Y_c = \frac{WL^4}{120EI}$
Fixed Beams				
	$S_A = \frac{W}{2}$	$M_c = \frac{WL}{8}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^3}{192EI}$
	$S_A = \frac{Wb^2}{L^3}(3a+b)$ $S_B = \frac{Wa^2}{L^3}(3b+a)$	$M_A = -\frac{Wab^2}{L^2}$ $M_B = -\frac{Wba^2}{L^2}$	$\theta_A = \theta_B = 0$	$Y_D = \frac{Wa^3b^3}{3EIL^3}$
	$S_A = \frac{WL}{2}$	$M_A = M_B = -\frac{WL^2}{12}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^4}{384EI}$
	$S_A = \frac{3WL}{20}$ $S_B = -\frac{7WL}{20}$	$M_A = -\frac{WL^2}{30}$ $M_B = -\frac{WL^2}{20}$	$\theta_A = \theta_B = 0$	$Y_{max} = 0.00131\frac{WL^4}{EI}$ $at\ x = 0.525L$
	$S_A = \frac{WL}{4}$	$M_A = M_B = -\frac{5WL^2}{96}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{0.7WL^4}{384EI}$

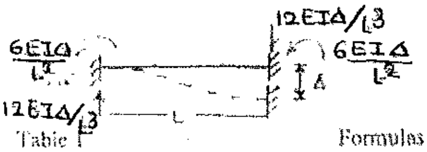
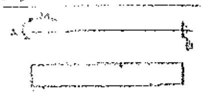
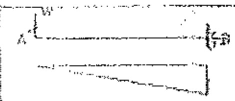
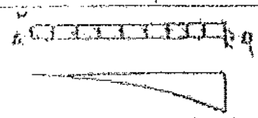
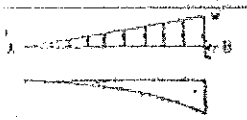
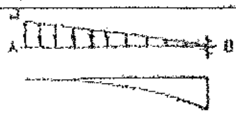
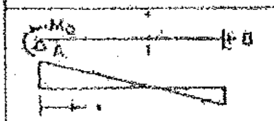
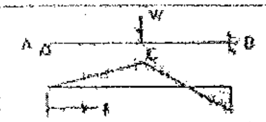
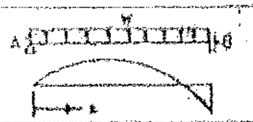
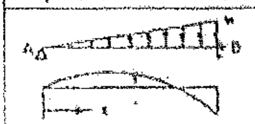
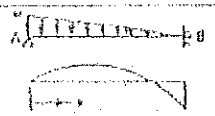


Table 1

Formulas for Beams

Structure	Shear	Moment	Slope	Deflection ↓
Cantilever Beam				
	0	M_o	$\theta_A = \frac{M_o L}{EI}$	$Y_A = \frac{M_o L^2}{2EI}$
	W	$M_B = -WL$	$\theta_A = -\frac{WL^2}{2EI}$	$Y_A = \frac{WL^3}{3EI}$
	$S_B = -WL$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{6EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{6}$	$\theta_A = -\frac{WL^3}{24EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{8EI}$	$Y_A = \frac{11WL^4}{120EI}$
Propped Cantilever				
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{M_o}{2}$	$\theta_A = -\frac{M_o L}{4EI}$	$Y_{max} = \frac{M_o L^2}{27EI}$ at $x = \frac{L}{3}$
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{3WL}{16}$ $M_c = \frac{5WL}{32}$	$\theta_A = \frac{WL^2}{32EI}$	$Y_{max} = 0.00932\frac{WL^3}{EI}$ at $x = 0.447L$
	$S_A = \frac{Wb^2}{2L^3}(a+2L)$ $S_B = -\frac{Wa}{2L^3}(3L^2 - a^2)$	$M_B = -\frac{Wab}{L^2}(a + \frac{b}{2})$	$\theta_A = \frac{Wab^2}{4EIL}$	$Y_D = \frac{Wa^3 b^2}{12EIL^3}(3L + a)$
	$S_A = +\frac{3WL}{8}$	$M_B = -\frac{WL^2}{8}$	$\theta_A = \frac{WL^3}{48EI}$	$Y_{max} = 0.0054\frac{WL^4}{EI}$ at $x = 0.422L$
	$S_A = +\frac{WL}{10}$	$M_{max} = 0.03WL^2$ at $x = 0.447L$ $M_B = -\frac{WL^2}{15}$	$\theta_A = \frac{WL^3}{120EI}$	$Y_{max} = 0.00239\frac{WL^4}{EI}$ at $x = 0.447L$
	$S_A = +\frac{11WL}{40}$	$M_{max} = 0.0423WL^2$ at $x = 0.329L$ $M_B = -\frac{7WL^2}{120}$	$\theta_A = \frac{WL^3}{80EI}$	$Y_{max} = 0.00305\frac{WL^4}{EI}$ at $x = 0.402L$