The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Mechanical Engineering

Study Programme	:	Bachelor of Technology Honours in Engineering
Name of the Examination	:	Final Examination
Course Code and Title	:	**DMX5533/ MEX5233 Dynamics of Mechanical Systems**
Academic Year	:	2019/20
Date	:	14th August 2020
Time	:	0930 hours -1230 hours
Duration	:	**3 hours**

General instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Eight (08)** questions in **Six (06)** pages.
3. Answer any **05** questions with the **compulsory (Question 01)** question.
4. Answer for each question should commence from a new page.
5. Relevant charts / codes are provided.
6. This is a Closed Book Test **(CBT).**
7. Answers should be in clear handwriting.
8. Do not use red colour pen.

Question 01 – *(20 Marks)* – **(Compulsory)**

The Figure Q01(a) shows a simple idealization of a force sensor, which is used to measure the force F, by providing an electrical signal. The electrical signal is proportional to the length 's' of the spring.

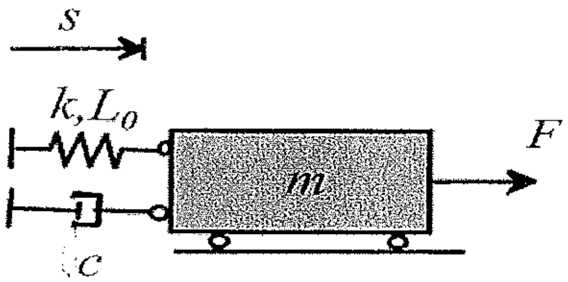

Figure Q01(a)

where; k is spring stiffness

L_0 is un-stretched length of spring

c is dashpot coefficient

m is mass

At time $t = 0$ the system is at rest, and $F = 0$. At time $t = 1 second$ a constant force of F=100N is applied to the mass m. The Figure Q01(b) shows the variation of s with time t for $0 < t < 5$ *seconds*.

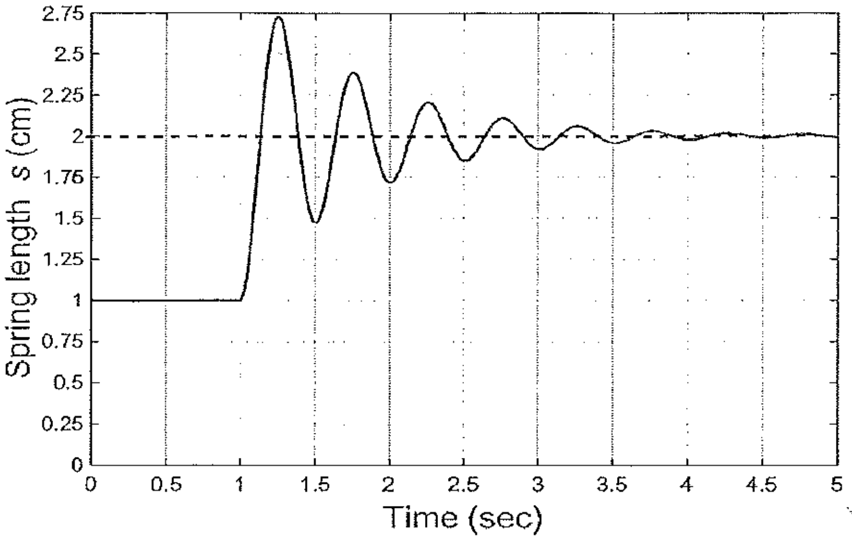

Figure Q01(b)

(a) Using the graph provided in (Figure Q01(b)), answers the following questions.

i. What is the period (T) of vibration?

ii. What is the damped natural frequency (ω_d) of the systems?

iii. Find the log decrement of the vibration δ (be careful to use the correct origin).

iv. Calculate the damping factor of the system (β).

v. Find the undamped natural frequency of the system (ω_n).

vi. Find the un-stretched length of the spring (L_0)

vii. Calculate the spring stiffness (k).

viii. What is the value of the mass (m)?

ix. Calculate the dashpot coefficient (c).

(b) The sensor is now used to measure a force that vibrates harmonically $F(t) = F_0 \sin\omega t$. The Figure Q1(c) shows the steady-state variation of the spring length s with time. Calculate the amplitude of the force F_0.

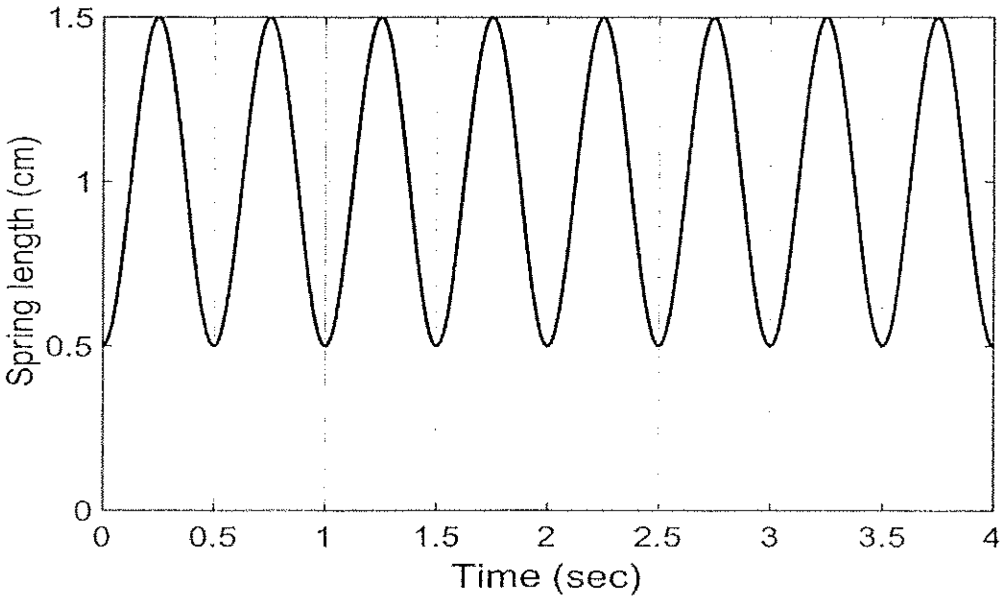

Figure Q1(c)

Note: The frequency of the force is equal to the natural frequency (the period of vibration is equal to the period in the Figure Q01(b)).

Question 02 - *(20 Marks)*

The pistons of the hydraulic system shown in Figure Q02, are massless and frictionless. The mass ***M*** is connected to piston ***Q*** of cross-sectional area A_2 by a rigid massless rod which slides in the frictionless bearings. The piston ***P*** has a cross sectional area A_1. The region between the pistons ***P*** and ***Q*** is filled with an incompressible fluid. If the constriction in the line connecting the two cylinders has a flow resistance ***R***, show that the transfer function relating *F* (force) to *Y* (deflection) is given by,

$$\frac{Y(s)}{F(s)} = \frac{K}{s(s+\tau)} \quad \text{where } K = \frac{A_2}{MA_1} \quad \text{and } \tau = \frac{RA_2^2}{M}$$

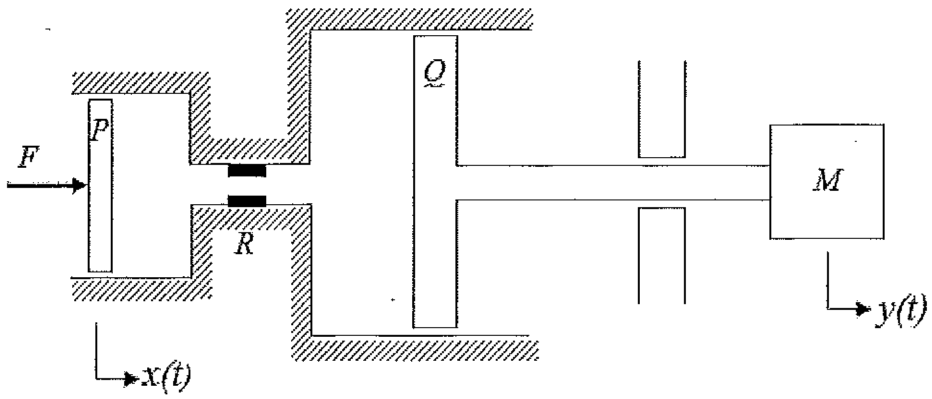

Figure Q02

Question 03 - *(20 Marks)*

a) A mass of **55kg** compressor rotor with a shaft stiffness of $\mathbf{1.4 \times 10^7 N/m}$, is operated at **6000 rpm** speed. The internal damping was provided to the compressor with a damping ratio of β = **0.05**. The rotor is assumed to have a worst-case eccentricity of **1000μm** (*e* = **0.001 m**). Calculate;
   i. The critical speed of the rotor.
   ii. The radial amplitude at given operating speed.
   iii. The whirl amplitude at the system's critical speed.

b) The clearance specification for the rotor inside the compressor housing limits the whirl amplitude at resonance to be **2mm**. If the whirl amplitude at operating speed is greater than the allowable clearance,
   i. what percent of change in mass is required to redesign this system?
   ii. what percent of change in stiffness would result in the same design?
   iii. Discuss the feasibility of such changes to the system.

Question 04 - *(20 Marks)*

A cantilever of length L and mass m per unit length has a vertical prop at the free end so that the fixed end and the free end of the cantilever are at the same level. The static deflection y of the propped cantilever at a distance x from the fixed end is given by

$$y = \frac{mg}{48EI}\left(2x^4 + 3L^2x^2 - 5Lx^3\right)$$

where EI is the flexural rigidity of the beam. Assuming that the cantilever takes the same form as the static deflection curve when it vibrates transversely, determine the natural frequency of transverse vibration of the propped cantilever in terms of ***EI*, *L*** and ***m***.

Question 05 - *(20 Marks)*

Compute the transfer function of the block diagram shown in Figure Q05 in following two cases.

(a) Reducing a system to single transfer function
(b) Label signals and draw a signal-flow graph for each of the block diagram shown in the Figure Q05.

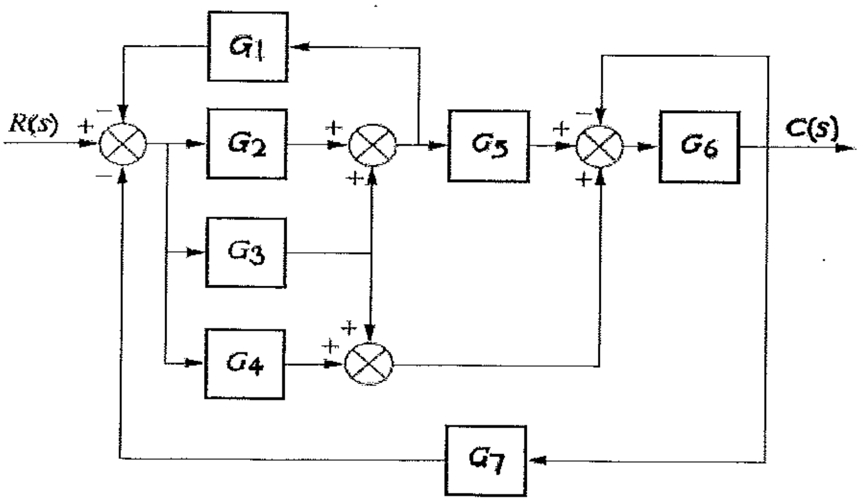

Figure. Q05

Question 06 - *(20 Marks)*

The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{K}{(s+1)(s^2+8s+20)}$$

where K is the gain of the controller.

i. Draw the root locus diagram as a function of K and determine the range of K for which the system is stable.
ii. Verify your result using Routh's analysis.
iii. If s = -3 is a closed loop pole of the system when K = 10, derive an expression for the response of the system for a unit impulse input when K = 10. Assume all initial conditions to be zero.

Question 07 - *(20 Marks)*

(a) A system controlling the position of a work piece can be modeled as shown in Figure Q07.

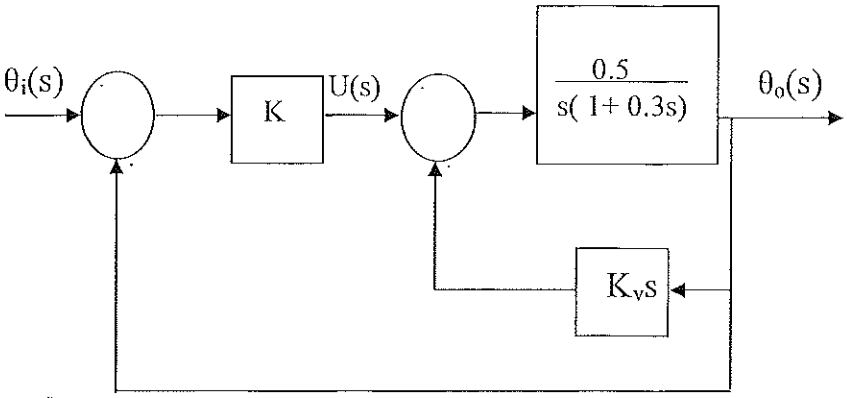

Figure Q07

i. Derive the closed loop transfer function $\theta_o(s)/U(s)$ of the inner loop in terms of K_v.

ii. Hence obtain the closed loop transfer function of the complete system in terms of **K** and K_v.

iii. Suggest appropriate values of **K** and K_v to obtain a closed loop response with a damping ratio **0.7** and an undamped natural frequency of **3 rad/s.**

(b) A system has a unit step response given by $c(t) = 1 - e^{-0.2t}$. Determine the unit impulse and the unit ramp response of the system. Assume that all initial conditions are zero.

Question 08 - *(20 Marks)*

Figure Q08 shows a control system. Draw a Bode diagram of the open loop transfer function and determine the value of the gain ***K*** such that the phase margin is **50°**. What is the gain margin of this system with this gain ***K***?

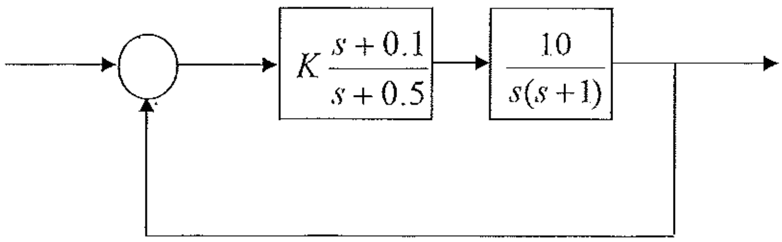

Figure Q08

LAPLACE TRANSFORMS

TIME FUNCTION f(t)	LAPLACE TRANSFORM F(s)
Unit Impulse $\delta(t)$	1
Unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{df(t)}{dt}$	$sF(s)-f(0)$
$\frac{d^n f(t)}{dt^n}$	$s^n F(s)-s^{n-1}f(0)-s^{n-2}\frac{df(0)}{dt}\ldots-\frac{d^{n-1}f(0)}{dt^{n-1}}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$