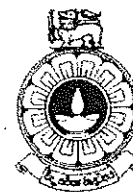


The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Electrical and Computer Engineering



Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	: ECX5233 / EEX5533 Communication Theory and Systems
Academic Year	: 2019/20
Date	: 11 th October 2020
Time	: 0930-1230 hrs.

General Instructions

1. Read all instructions carefully before answering the questions.
 2. This question paper consists of **Eight (8)** questions in **Six (6)** pages.
 3. Answer any **Five (5)** questions only. All questions carry equal marks.
 4. Answer for each question should commence from a new page.
 5. Relevant charts / codes are provided.
 6. This is a Closed Book Test (**CBT**).
 7. Answers should be in clear handwriting.
 8. Do not use red color pen.
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1.

(a)

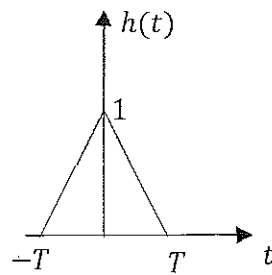


Fig. 1(a)

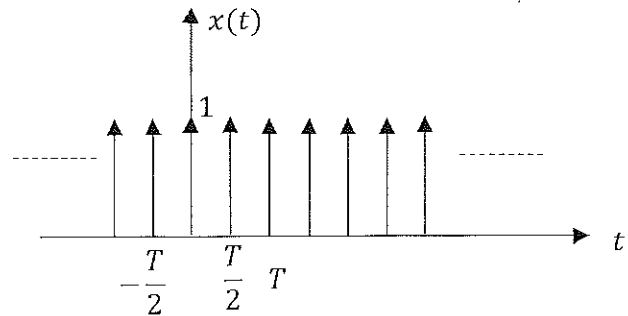


Fig. 1(b)

The impulse response $h(t)$ of an LTI system is shown in Fig.1(a). $h(t)$ can be found by convolving a rectangular pulse $p(t)$ with itself. i.e. $h(t) = p(t) * p(t)$

- (i) Find the pulse-width of $p(t)$. [2 marks]
 - (ii) Find the transfer function $H(\omega)$ of the system. [5 marks]
 - (iii) If the impulse train $x(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\frac{T}{2})$ shown in Fig. 1(b) is applied as the input to the system find and sketch the system output $y(t)$. [5 marks]
 - (iv) What are the frequencies available in the output signal $y(t)$? [2 marks]
- (b) (i) Express $x(t)$ shown in Fig. 1(b) as a complex Fourier series and find Fourier coefficients C_n . [4 marks]
- (ii) Sketch $|C_n|$ vs. $n\omega_0$. [2 marks]

2.

(a)

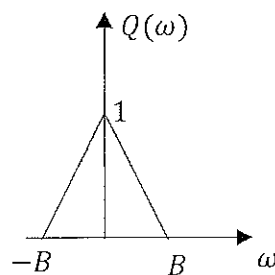


Fig. 2(a)

The signal $q(t)$ has a frequency spectrum shown in Fig. 2(a). The signal shown in Fig.1 (b) is used to sample the signal $q(t)$. The sampled signal $q'(t)$ is given by $q'(t) = q(t) \cdot x(t)$.

- (i) Show that $q'(t)$ can be written as $q'(t) = \sum_{n=-\infty}^{\infty} C_n q(t) e^{jn\omega_0 t}$, where C_n is a function of n , where $\omega_0 = \frac{4\pi}{T}$. [3 marks]
- (ii) Using the expression in 2(a)(i) derive an expression for $Q'(\omega)$, where $Q'(\omega)$ is the Fourier transform of $q'(t)$. [6 marks]
- (iii) Sketch $Q'(\omega)$. Assume $C_n = 1$. [2 marks]
- (iv) Using the sketch of 2 (a) (iii), find the maximum possible value for T which will enable us to retrieve $q(t)$ from $q'(t)$, without any loss of information. [4 marks]

(b)

Three audio channels, each having a bandwidth of 4 kHz are to be frequency division multiplexed to form a baseband signal. The resulting signal is discretized using an impulse train. What is the maximum possible sampling time T_s ? [5 marks]

3.

(a)

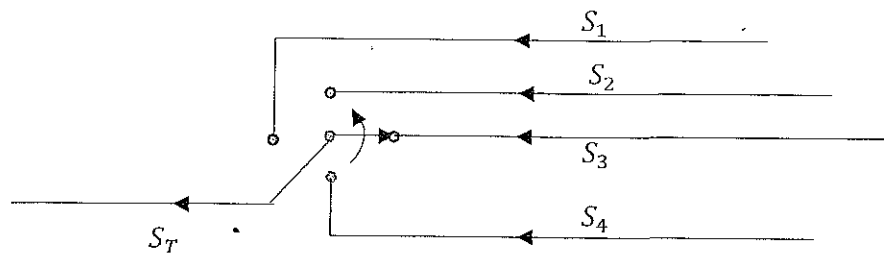


Fig. 3

Four sinusoidal signals S_1, S_2, S_3 and S_4 are time division multiplexed into a composite signal S_T using a high speed electronic switch / sampler as shown in the Fig. 3. The highest frequency component available in each of the analog signals are as follows:

S_1 - 4 kHz; S_2 - 16 kHz; S_3 - 12 kHz and S_4 - 8 kHz

The multiplexer outputs the data at a rate of 100 kilo samples per second.

At the receiver, the signal S_T is demultiplexed back to S_1, S_2, S_3 and S_4 . It was observed that some of the signals are distorted after demultiplexing process.

What are the distorted signals? Justify your answer.

[8 marks]

(b)

Four TV channels are frequency division multiplexed and converted into a single base band signal. The bandwidth of each channel is 7.5 MHz.

- (i) Sketch the frequency spectrum of the baseband signal. [3 marks]
- (ii) Amplitude modulation is used in the preparation of the baseband signal.

1. What are the subcarrier frequencies to be used in the preparation of the baseband signal? [4 marks]
2. Explain how the baseband is prepared using amplitude modulation technique. [5 marks]

4.

(a)

In the PAL (phase alternating line) system one picture frame consists of two half frames. The first frame consists of the line 1, line 3, line 5,etc. . The second frame consists of line 2, line 4, line 6 etc. The horizontal deflection signal and the vertical deflection signal of a TV receiver is shown in Fig. 4 (a) and Fig. 4 (b) respectively.

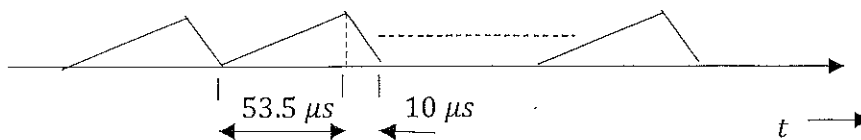


Fig. 4 (a)

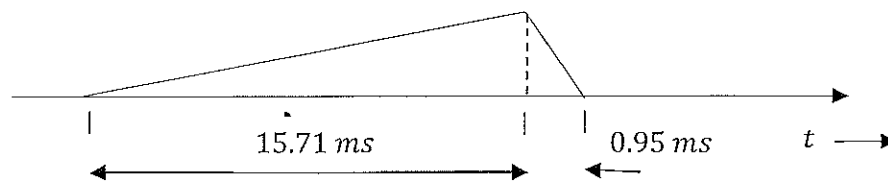


Fig. 4 (b)

- (i) Using Fig. 4(a) and Fig. 4(b) calculate the number of active lines per picture frame on the TV receiver. [6 marks]
- (ii) In Fig. 4 (b) the duration of the second portion of the sawtooth signal is indicated as 0.95 ms. How does the electron beam of the picture tube behave during this period? [3 marks]

(b)

The triangular signal $m(t)$ shown in Fig. 4 (c) is used as the modulating signal for a phase modulator.

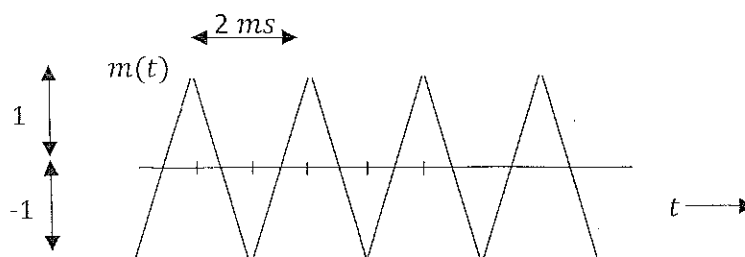


Fig. 4(c)

- (i) Write an expression for the instantaneous phase $\theta(t)$ of the phase-modulated signal. The carrier frequency is $\omega_c \text{ rad/s}$. Assume that $\theta(t)$ varies linearly with $m(t)$. Assume that the phase modulation constant is k_p . [3 marks]
- (ii) Derive an expression for the instantaneous frequency ω_i for the phase-modulated signal given in 4 (b) (i). [3 marks]
- (iii) If the carrier frequency f_c is 10 MHz and the constant k_p is 10π , write an expression for the modulated carrier. Assume that the carrier signal is a cosine signal and the modulating signal is as given in Fig. 4(c). [5 marks]

5.

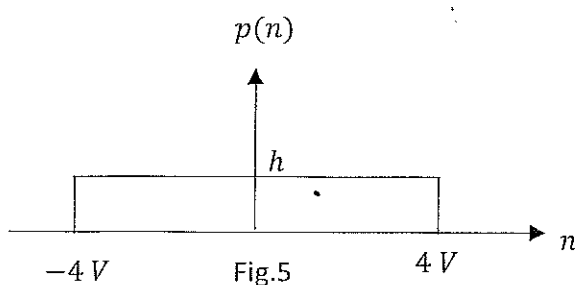
(a)

A pulse code modulated signal has 7 data bits and one parity bit. During the transmission some of the data bits change the value due to medium noise.

- (i) Find the probability for a 3-bit error. [5 marks]
- (ii) Find the total error probability. [5 marks]

Assume that the bit error probability $p_e = 0.001$.

(b)



A binary data source transmits $+2 \text{ V}$ or -2 V with equal probability. $+2 \text{ V}$ is considered to be a '1' and -2 V is considered to be a '0'. During the transmission noise is added to the signal.

If the received signal amplitude is greater than or $+1.5 \text{ V}$, the transmitted bit is assumed to be a '1'. If it is less than $+1.5 \text{ V}$ the corresponding bit is assumed to be a '0'. The probability density function of noise is shown in Fig.5

- (i) Find the probability that '1' is received as a '0'. [5 marks]
- (ii) Find the total error probability. [5 marks]

6.

(a)

What is the relationship between the information content of a message and the probability of occurrence of that message? [3 marks]

(b)

A speaker describes one of his experiences during the second world war using 2000 words. These 2000 words come out from the speaker's vocabulary of 12000 words. Assuming that each of the 12000 words of the vocabulary is equally likely to occur, determine the amount of information in the speech.

[7 marks]

(c)

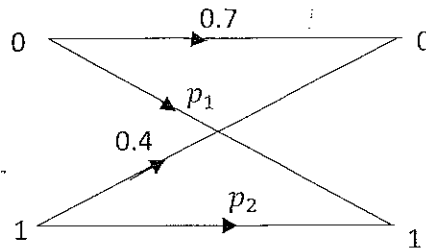


Fig. 6

A binary symmetric channel is shown in Fig.6.

(i) Find the values of p_1 and p_2 . [3 marks]

(ii) Determine the channel matrix if two of the above channels are cascaded. [7 marks]

7.

(a)

(i) Define the autocorrelation function $\mathcal{R}_x(t_1, t_2)$ of a random process. [3 marks]

(ii) How do you find whether a given random process is a WSS (wide sense stationary) process using $\mathcal{R}_x(t_1, t_2)$? [4 marks]

(b)

A random process is given by $x(t) = A \cos(\omega_0 t + \Theta)$, where A and ω_0 are constants. Θ is a random variable distributed over the interval $(0, 2\pi)$.

(i) Find the autocorrelation function $\mathcal{R}_x(t_1, t_2)$. [6 marks]

(ii) Find whether the process is a WSS process. [4 marks]

(iii) Find the power of the signal. [3 marks]

8.

During the transmission of a signal channel noise is added to the signal. When a filter is used to remove this noise, original signal is distorted due to filtering and additional distortion is added to the signal as the distortion noise.

If the power spectral densities of the signal and the noise at the receiver input are $S_m(\omega)$ and $S_n(\omega)$ respectively total noise at receiving filter output can be written as

$$N_0 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\left| H(\omega) - \frac{S_m(\omega)}{S_m(\omega) + S_n(\omega)} \right|^2 (S_m(\omega) + S_n(\omega)) + \frac{S_m(\omega) S_n(\omega)}{S_m(\omega) + S_n(\omega)} \right] d\omega$$

where $H(\omega)$ is the transfer function of the filter.

- (a) Find the optimum value of $H(\omega)$ for minimum output noise. [8 marks]
- (b) Write an expression for the minimum value of N_0 . [5 marks]
- (c) Find and sketch the impulse response of the optimum filter if $S_m(\omega) = \frac{4}{8 + \omega^2}$ and $S_n(\omega) = 4$. [7 marks]

Some important Fourier Transform pairs

$g(t)$	$G(\omega)$
$ t $	$-\frac{2}{\omega^2}$
$\delta(t)$	1
$e^{-a t }$	$\frac{2a}{a^2 + \omega^2}$