

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Mechanical Engineering



Study Programme : Bachelor of Technology Honours in Engineering
Name of the Examination : Final Examination
Course Code and Title : **DMX4572/MEX4272 Vibration and Fault Diagnosis**
Academic Year : 2019/20
Date : 17th October 2020
Time : 0930-1230hrs
Duration : **3 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Eight (8)** questions in **Six (6)** pages.
3. Answer any **Five (5)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. This is a Closed Book Test (CBT).
6. Answers should be in clear hand writing.
7. Do not use Red colour pen.

Question 01

[20 marks]

The inverted pendulum may be used as a simple model in the stability studies of inherently unstable systems such as rockets. Consider an inverted pendulum of point mass m and length l , which is restrained at its pivot (assumed to be smooth) by a torsional spring of stiffness k . This arrangement is sketched in Figure Q1.

- a) Derive an equation of motion for this system.
- b) Obtain an expression for the natural frequency of small oscillations θ about the vertical configuration. Under what conditions would such oscillations not be possible (i.e., the system would become unstable)?

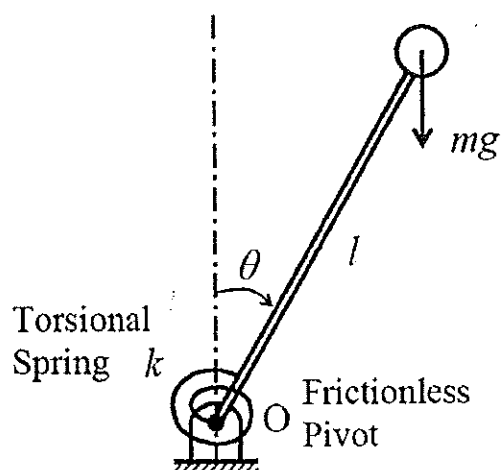


Figure Q1

Question 02

[20 marks]

- a) Consider the simplified model of a single-degree-of-freedom robot arm (single link) shown in Figure Q2. The link is driven by a dc motor, through a light shaft of torsional stiffness k_s . Discuss how mechanical vibrations in a robot arm could adversely affect its performance.

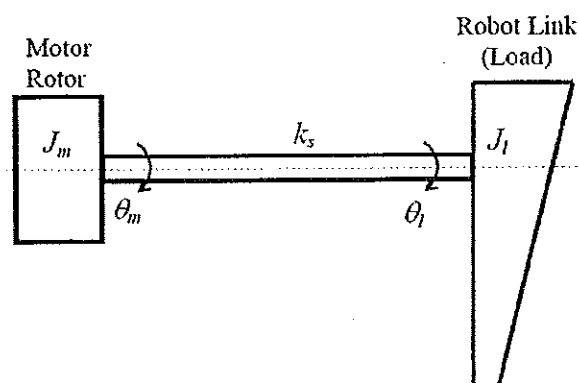


Figure Q2

- b) Find the total response of a viscously damped single *DoF* system subjected to a harmonic base excitation for the following data:

$$m = 10\text{kg}, \quad c = \frac{20\text{Ns}}{\text{m}}, \quad k = \frac{4000\text{N}}{\text{m}}, \quad y(t) = 0.05 \sin 5t \text{ m}, \quad x_0 = 0.02\text{m},$$

$$\dot{x}_0 = 10\text{m/s}$$

Question 03

[20 marks]

Figure Q3 shows a simple model of a motor vehicle that can vibrate in the vertical direction while traveling over a rough road. The vehicle has a mass of 1200 kg . The suspension system has a spring constant of 400 kN/m and a damping ratio of $\zeta = 0.5$. If the vehicle speed is 20 km/h , determine the displacement amplitude of the vehicle. The road surface varies sinusoidal with an amplitude of $Y = 0.05$ and a wavelength of 6 m .

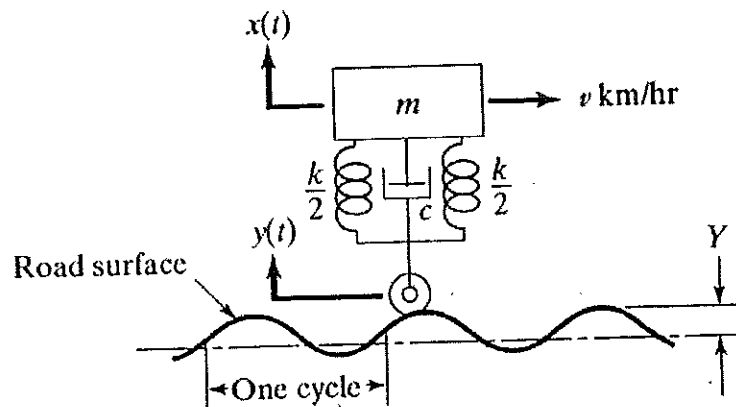


Figure Q3

Question 04

[20 marks]

- Define these terms: *cycle*, *amplitude*, *phase angle*, *linear frequency*, *period*, and *natural frequency*.
- How can we obtain the *frequency*, *phase*, and *amplitude* of a harmonic motion from the corresponding rotating vector?
- What is the difference between a *discrete* and a *continuous* system? Is it possible to solve any vibration problem as a discrete one?
- Describe three different ways of expressing a periodic function in terms of its harmonics.

Question 05**[20 marks]**

- a) Explain why mechanical vibration is an important area of study for engineers.
- b) Mechanical vibrations are known to have harmful effects as well as useful ones. Briefly describe five practical examples of “good vibrations” and five practical examples of “bad vibrations”.
- c) Under some conditions it may be necessary to modify or redesign a machine with respect to its performance under vibrations.
 - i. Describe the possible reasons for this.
 - ii. Explain the modifications that could be carried out on a machine in order to suppress its vibrations.

Question 06**[20 marks]**

A heavy machine, weighing $3000N$, is supported on a resilient foundation. The static deflection of the foundation due to the weight of the machine is found to be $7.5cm$. It is observed that the machine vibrates with an amplitude of $1cm$ when the base of the foundation is subjected to harmonic oscillation at the undamped natural frequency of the system with an amplitude of $0.25cm$.

- a) Determine the damping constant of the foundation.
- b) Find the dynamic force amplitude on the base.
- c) Find the amplitude of the displacement of the machine relative to the base.

Question 07**[20 marks]**

Find the natural frequencies and mode shapes of a spring-mass system, shown in Figure Q7, which is constrained to move in the vertical direction only. Take $n = 1$.

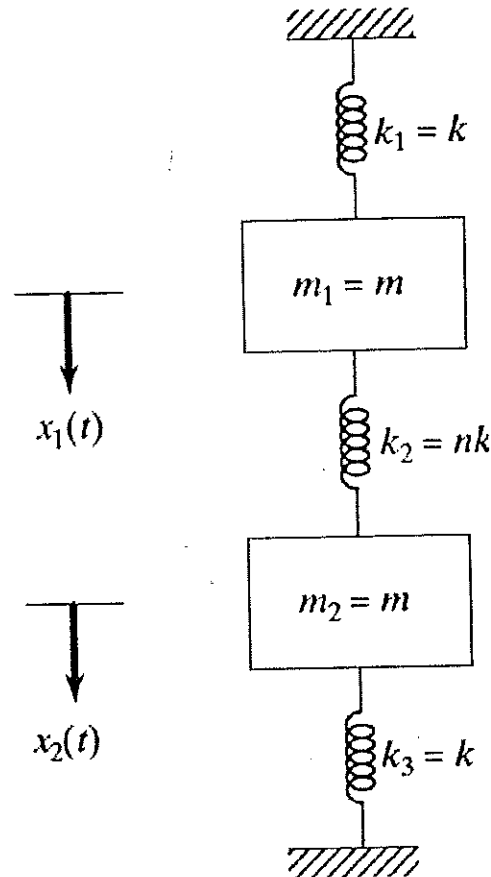


Figure Q7

Question 08**[20 marks]**

A mechanical system that is at rest is subjected to a unit step input $u(t)$. Its response is given by following equation. Determine the following.

$$y(t) = 2 - e^{-t}(2\cos t + \sin t)$$

- Transfer function of the system.
- Input-Output differential equation of the system.
- Damped natural frequency, undamped natural frequency, and the damping ratio.
- Response of the system to a unit impulse input.
- Steady state response for a unit step input.

Laplace transforms:

| TIME FUNCTION $f(t)$ | LAPLACE TRANSFORM $F(s)$ |
|--------------------------|--|
| Unit Impulse $\delta(t)$ | 1 |
| Unit step | $\frac{1}{s}$ |
| t | $\frac{1}{s^2}$ |
| t^n | $\frac{n!}{s^{n+1}}$ |
| $\frac{df(t)}{dt}$ | $sF(s) - f(0)$ |
| $\frac{d^n f(t)}{dt^n}$ | $s^n F(s) - s^{n-1} f(0) - s^{n-2} \frac{df(0)}{dt} \dots - \frac{d^{n-1} f(0)}{dt^{n-1}}$ |
| e^{-at} | $\frac{1}{s+a}$ |
| te^{-at} | $\frac{1}{(s+a)^2}$ |
| $\sin \omega t$ | $\frac{\omega}{s^2 + \omega^2}$ |
| $\cos \omega t$ | $\frac{s}{s^2 + \omega^2}$ |
| $e^{-at} \sin \omega t$ | $\frac{\omega}{(s+a)^2 + \omega^2}$ |
| $e^{-at} \cos \omega t$ | $\frac{s+a}{(s+a)^2 + \omega^2}$ |

END