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THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Bachelor of Technology Honors in Engineering / Bachelor of Software Engineering Honors

Final Examination (2019/2020)
MHZ4340 /MHZ4360/ MPZ4140 /MPZ4160: Discrete Mathematics I

Date: 28th July 2020 (Tuesday) | **Time: 13:30 – 16:30**

Instruction:

- **Answer only six questions.**
- **Please answer a total of six questions choosing at least one from each single section.**
- **All symbols are in standard notation and state any assumption that you made.**

SECTION – A

Q1.

I. Decide which of the following are propositions. What are the truth values of those that are propositions? [20%]

a) "$x^2 \geq 10$" ;
b) "5 + 7 = 1 0";
c) "Colombo is in England or $1 + 9 = 8$".
d) "London is the capital of India".

II. State the "convers", "inverse", and "contrapositive" of each of the following statement: [30%]

a) If x is an integer and x^2 is odd, then x is odd;
b) If 11 pigeons live in 10 birdhouses, then there are two pigeons that live in the same birdhouse.

III. Let p, q, and r be three statements.
Verify that each of the following statement is a tautology or not. [30%]

a) $\big((p \to q) \to r\big) \to \big(p \to (q \to r)\big)$
b) $\big(p \to (r \vee q)\big) \to \big((p \to r) \vee (p \to q)\big)$

IV. Determine the truth value and Negation of the each of the following statements: [20%]

a) $\forall x \in \mathbb{R},\ (x^2 \geq x)$;
b) $\exists x \in \mathbb{R}, (x = 1)$.

Q2.

I. Test the validity of the following arguments:

a) If I want to be a lawyer, then I want to study logic.
If I don't want to be a lawyer, then I don't like to argue.

Therefore, If I like to argue, then I want to study logic. **[30%]**

b) If Mohan goes to town, then Melani stays at home.
If Melani does not stay at home, Rita will cook.
Rita will not cook.

Therefore, Mohan does not go to town. **[30%]**

II. By using truth tables, prove De.Morgan laws of propositions. **[20%]**

III. Prove directly that the "if n is an odd integer, then n^2 is odd". **[20%]**

Q3.

I. Show that $\sqrt{2}$ is an irrational number. **[20%]**

II. Using Mathematical induction, for a positive integer n, prove each of the following: **[60%]**

a) $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$ for all $n \geq 1$;

b) $2^{3n+1} + 5$ is divisible by 7.

III. By giving a counter example, disprove each of the following statements:

a) all odd number are divisible by 3; **[10%]**

b) the square root of any integer is irrational. **[10%]**

SECTION – B

Q4.

I. Write down the elements in each of the following set: **[20%]**

a) $A = \{x : x^4 - 5x^2 + 6 = 0,\ x \in \mathbb{R}^+\}$;
b) $B = \{x : x \geq 14,\ x = 2n + 1,\ n \in \mathbb{Z}^+\}$;
c) $C = \{x : x^2 + 1 = 10,\ x \in \mathbb{N}\}$,
d) $D = \{x : |x + 3| \leq 6,\ x \in \mathbb{Z}^+\}$.

II. $L = \{9, 19, 29, 39, 49\}$, $M = \{39, 49, 59, 69\}$, $N = \{59, 69, 79, 89\}$. Find

a) $L \oplus M$;
b) $M \oplus N$;
c) $L \cap (M \oplus N)$, where $\oplus$ is symmetric difference. **[15%]**

III.

a) Define the Cartesian product of set A and B. **[05%]**
b) $A = \{a, b, ab, ba\}$ and $B = \{1,\ 12, 21\}$. Find $A \times B$ and A^2. **[20%]**

IV. Let $E = \{e, d,$ ed, $de\}$. Find the power set $P(E)$ of E. **[10%]**

V. Without using Venn diagram, Show that

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. **[30%]**

Q5.

I. Let $h: \mathbb{R} \to \mathbb{R}$ be defined by

$$h(x) = \begin{cases} x + 7\ ; & x \leq 0 \\ -2x + 5; & 0 < x < 3\ . \\ x - 1; & x \geq 3 \end{cases}$$

Find $h(-3)$, $h(0)$, and $h(3)$. **[30%]**

II. Let $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = ax^2 + b$ and $g(x) = cx^2 + d$ respectively for all $x \in \mathbb{R}$, where $a, b, c,$ and d are constants with $a \neq 0$ and $c \neq 0$.
Find the relationship(s) between the constant a, b, c, d, $if\ f \circ g(x) = g \circ f(x)$ for all x. **[30%]**

III. Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Define $m(x) = \frac{x-2}{x-3}$. Prove that $m(x)$ is invertible and find a formula for $m^{-1}(x)$. **[40%]**

Q6.

I. Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5\}$. In each of the following, find all the pairs of A x B that belong to R_1 and R_2.

a) $R_1 = \{(x, y) | \, x \geq y; \; x \in A, y \in B\}$ **[10%]**

b) $R_2 = \{(x, y) | \, x^2 = y; x \in A, y \in B\}$. **[10%]**

II.

a) Define the equivalence relation by the usual notation. **[10%]**

b) Determine whether the following relations are equivalence relation or not.

α) Let A be a set if integers and R_3 be the relation of $A \times A$ defined by $(a, b)R_3(c, d)$ if $ad = bc$. **[25%]**

β) If R_2 be the relation which is defined by " aR_2b iff $a - b$ is divisible by 10" for $a, b \in \mathbb{Z}$. **[25%]**

III. Show that "$x \leq y$" is a partial order relation in $\mathbb{R}$, where $x, y \in \mathbb{R}$. **[20%]**

SECTION – C

Q7.

I. Let a, b, and c be any integer numbers. Prove that, **[45%]**

a) if $a|b$ and $a|c$, then $2a| \, (4b + 6c)$,

b) if $a|b$, and $b|c$, then $a|c$,

c) If $a|b$ and $b|a$, then $a = \pm b$.

II. Let $x, y \in \mathbb{Z}$. If $6|(3x - y^2)$, then show that $3|(3x^2 - 3xy - xy^2 + y^3 + 24x)$. **[20%]**

III. If $\gcd(a, m) = \gcd(b, m) = 1$, then show that $\gcd(ab, m) = 1$. **[20%]**

IV. Show that $\gcd(na, nb) = n \gcd(a, b)$, for any positive integer n. **[15%]**

Q8.

I. Let a and b be integers. Show that $gcd(a,b) = gcd(a+3b, 2b)$. **[10%]**

II. Let a and b are integers and $gcd(a,b) = 1$. Prove that $gcd(ac,b) = gcd(c,b)$, where $a,b,c \in \mathbb{Z}$. **[15%]**

III. Show that if a and b are relatively prime numbers, then $\gcd(2a+3b, 3a+2b) = 1$or 5. **[30%]**

IV. Use the Euclidean algorithm to find the greatest common divisor of 3356 and 246 and express it in terms of the two integers. **[25%]**

V. Either find all solutions or prove that there are no solutions for the Diophantine equation $2080x + 347y = 5$. **[20%]**

Q9.

I Let a, b, c and d denote integers. Let m be a positive integer. Show that:

a) If $a \equiv b(mod\ m)$ and $b \equiv c(mod\ m)$, then $a \equiv c(mod\ m)$. **[10%]**

b) If $a \equiv b(mod\ m)$, $c \equiv d(mod\ m)$, and $e \equiv f(mod\ m)$, then $(3a - 2c + 5e) \equiv (3b - 2d + 5f)(mod\ m)$. **[20%]**

c) If $a \equiv b(mod\ m)$ and $d|m, d > 0$, then $a \equiv b(mod\ d)$. **[15%]**

II Solve the following system of congruence: **[55%]**

$x \equiv 2 \pmod{5}$

$x \equiv 5 \pmod{11}$

$x \equiv 7 \pmod{13}$

$x \equiv 3 \pmod{17}$

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