The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Mechanical Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: DMX6573 Advanced control systems**
Academic Year	: 2020/21
Date	: 12th January 2022
Time	: 14:00-17:00hrs
Duration	**: 3 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Seven (7)** questions in **Seven (7)** pages.
3. Answer any **Five (5)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. This is a Closed Book Test (CBT).
6. Answers should be in clear handwriting.
7. Do not use Red colour pen.

Q1

Consider the electrical circuit shown in figure Q1. The dynamic system's governing equations derived from nodal analysis are given below.

$$C_1\frac{de_1}{dt}+\frac{e_1-e_2}{R_3}+\frac{e_1-e_0}{R_1}=0$$

$$C_2\frac{de_2}{dt}+\frac{e_2-e_1}{R_3}+\frac{e_2-e_0}{R_2}=0$$

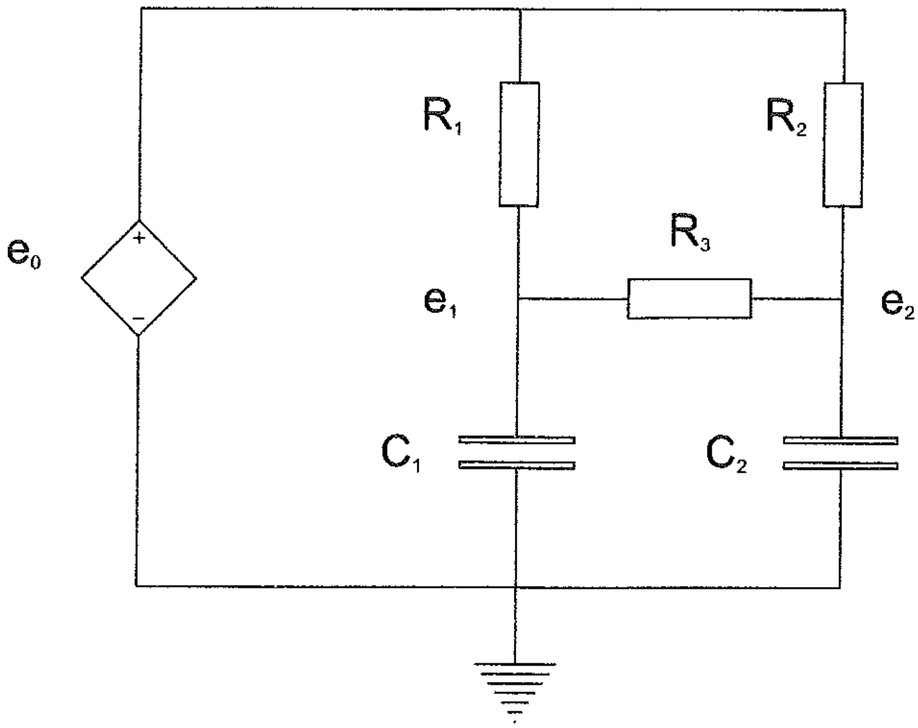

Figure Q1

i) Derive the state-space model for the electrical circuit shown in figure Q1. Please note that $C = [0 \quad 1]$, $and\ D = 0$ in the state-space model. [8 Marks]

ii) If $R_1C_1 = R_2C_2$;

   a. Then find the behavior of the system considering controllability and observability matrix. [8 Marks]

   b. If $R_1 = R_2 = R_3 = 1\Omega$ and $C_1 = C_2 = 1\text{F}$, determine the roots of the characteristic equation and find the transfer function of the system. Then based on the data available, comment on the behavior of the system. [6 Marks]

Q2

i) The single input linear-time invariant system with n^{th} order state differential equation is represented by;

$$\dot{X}(t) = AX(t) + B\,U(t)$$

and state-feedback law is given by;

$$U(t) = -KX(t)$$

a) Derive the state differential equation for the closed-loop system. [4 Marks]

b) Sketch the control configuration for the state regulator. [2 Marks]

ii) Figure Q2 shows a satellite, which requires attitude control so that antennas, sensors, and solar panels are correctly oriented. The angle θ describes that the satellite's orientation was measured with respect to the inertial reference, with no angular acceleration. The control signals come from the reaction jets that produce a torque $\tau(t)$ (=Fd) about the mass centre.

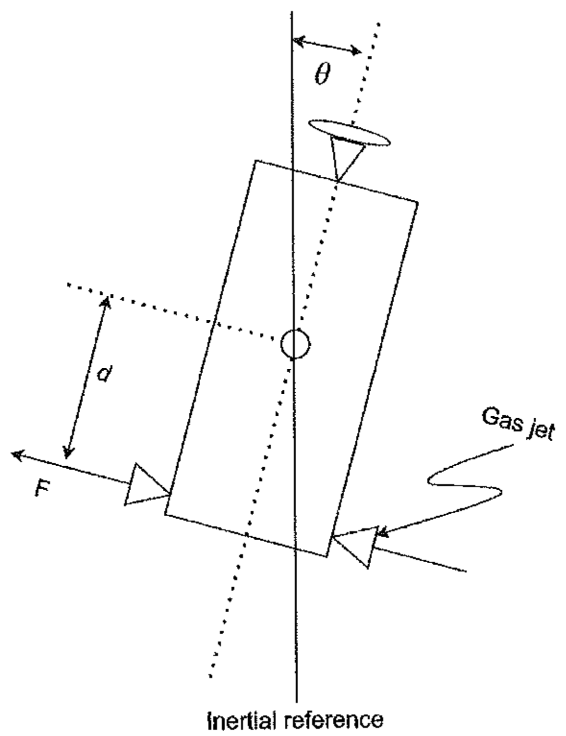

Figure Q2

The satellite is assumed to be in a frictionless environment. If $\tau(t)$ is the system input, and $\theta(t)$ is the system output;

$$\tau(t) = J\frac{d^2\theta(t)}{dt^2}$$

Where j is the moment of inertia of the satellite;

$$u(t) = \frac{\tau(t)}{J}$$

a) Show the state equation as $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. [2 Marks]

b) Derive the characteristic equation of the closed-loop system. [4 Marks]

c) The closed-loop system poles needed to position at the $-4 \pm j4$, determine the feedback matrix of the system. [4 Marks]

d) Sketch the control configuration for the state regulator for the satellite control system.

[4 Marks]

Q3

The governing equation of the spring-mass-damper system and the total energy $V(x_1, x_2)$ is given below

$$\ddot{x_1} + B\dot{x_1} + Kx_1 = 0 \ \ ; \dot{x}_1 = x_2$$

$$V(x_{1,}x_2) = K\frac{1}{2}x_1^2 + \frac{1}{2}x_2^2$$

i) State the behavior of the system [5 Marks]

ii) Derive the rate of change of the energy function as $\dot{V}(x_1, x_2) = -Bx_2^2$. [5 Marks]

iii) State the stability of the system under positive damping (B>0). [5 Marks]

iv) State the stability of the system under zero damping (B=0). [5 Marks]

Q4

i) A fuzzy set **A** in the universe **R** whose membership function is given by

$\mu_A(x) = 2 - |x-2|$ for $|x-2| \le 1$

= 0 otherwise

a) Sketch the membership function [4 marks]

b) What is α-cut of A for α=0.6? [4 marks]

ii) Two fuzzy sets A and B, are represented by the membership functions shown in figure Q4. Sketch the membership functions of the following fuzzy sets relations.

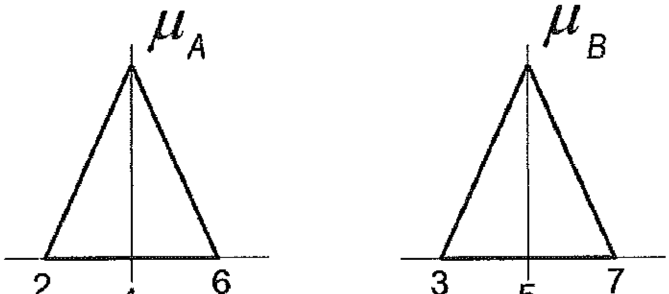

Figure Q4

a) $A \cup B$ [4 Marks]

b) $A \cap B$ [4 Marks]

c) $\bar{A}$ [4 Marks]

Q5

The θ is the deviation angle of the pendulum from vertical in the clockwise, and τ is the torque applied to the pendulum, which is shown in figure Q5. The state variable is defined as $x_1 = \theta$, and $x_2 = \dot{\theta}$. The discrete-time state-space equation of the inverted pendulum is given below. The universe of discourse for both variables are $-2^o \le x_1 \le 2^o$, and $-5dps \le x_2 \le 5dps$ (dps= degrees per second). The output ***u(k)*** has five membership functions, which is and $-24 \le u(k) \le 24$.

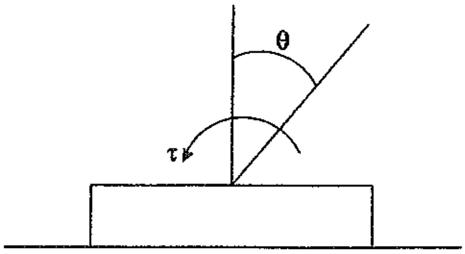

Figure Q5

$$x_1(k+1) = x_1(k) + x_2(k)$$

$$x_2(k+1) = x_1(k) + x_2(k) - u(k)$$

i)

a) Define three membership functions for x_1 for values of positive, zero, and negative. [2 Marks]

b) Define three membership functions for x_2 for values of positive, zero, and negative. [2 Marks]

c) Define five membership functions for u(k) for positive large, positive, zero, negative, and negative large. [2 Marks]

ii) The rule base is given in table Q5.

Table Q5

x_1 / x_2	P	Z	N
P	PL	P	Z
Z	P	Z	N
N	Z	N	NL

a) If initial conditions are $x_1(0) = 1^0$ and $x_2(0) = -4dps$, Find which rules are fired. [8 Marks]

b) Determine the defuzzified output of the system. [4 Marks]

Q6

i) Sketch the artificial neuron and label each component of the artificial neuron.[4 Marks]

ii) Derive the mathematical function for the artificial neuron. [4 Marks]

iii) The basic equation for the gradient descent training is

$$\Delta w_{ij} = -\eta \frac{\partial E}{\partial w_{ij}}$$

a) Explain how weights are updated in the neural network. [6 Marks]

b) What does the learning rate do in gradient descent training algorithms? [6 Marks]

Q7 The single neuron is shown in figure Q7.

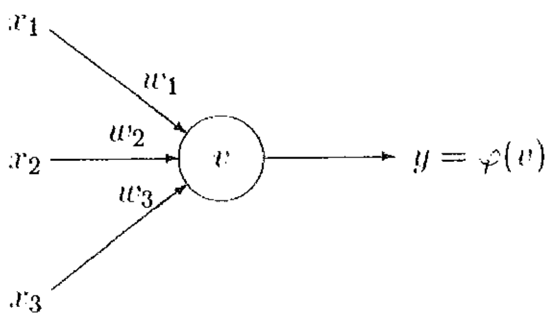

Figure Q7

The artificial neuron has three inputs x = (x_1, x_2, x_3) that receive only binary signals (either 0 or 1). Suppose that the weights corresponding to the three inputs have the following values $w_1 = -3, w_2 = 1, and w_3 = 2$, and the activation function is :

$$\varphi(v) = \begin{cases} 1 & if\ v \geq 2 \\ 0 & otherwise \end{cases}$$

i) Test how an artificial neuron works for all the input signals. [10 Marks]

ii) Suggest how to change the weights or/and the threshold level of this single unit in order to implement the logical exclusive OR gate function. [10 Marks]

End