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The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Mechanical Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: *Final Examination*
Course Code and Title	**: DMX6578 – Fluid Mechanics**
Academic Year	: 2020/2021
Date	: 30th January 2022
Time	: 14.00-17.00hrs
Duration	: **3 hours**

General Instructions

1. Read all the instructions carefully before answering the questions.
2. This question paper consists of 8 questions. All questions carry equal marks.
3. Answer **any 5** questions only.
4. Take acceleration due to gravity as **9.81 N/kg** and the density of water as **1000 kg/m^3** respectively where necessary.

Q1).

a) By considering the rotation of a fluid element, derive an expression for the rotation vector (ω).

b) The velocity distribution for a three dimensional flow is given by,

$$V = (a + by - cz)i + (d - bx - ez)j + (f + cx - ey)k$$

Where, a, b, c, d, e and f are arbitrary constants.

I. Show that the flow satisfies the continuity equation.

II. Does the velocity vector (V) represent irrotational flow? if not, determine the vorticity and rotation.

Q2).

a) What is the importance of potential flow theory?

b) In the context of potential flow theory, what is the importance of Laplace equation?

c) A source and sink of strength 4 m^2s^{-1} and 8 m^2s^{-1} are located at (-1,0) and (1,0) respectively. Determine the velocity and stream function at a point P(1,1) which is on the flownet of the resultant stream line.

Q3).

a) What do you understand by boundary layer thickness?
b) A velocity profile of a boundary layer on a flat plate is given by,

$$\frac{u}{U_0} = \left(\frac{y}{\delta}\right)^{\frac{1}{m}}$$

I. Derive the expressions for the displacement thickness (δ^*) and the momentum thickness (θ).

II. Show that the frictional drag (F_d) force per unit area due to the boundary layer is given by.

$$F_d = \frac{\rho U_0^2 m \delta}{(m+1)(m+2)}$$

Where, ρ=density of the fluid, U_0=free-stream velocity and δ=boundary layer thickness

Q4).

a) By considering the velocity field $V(x,y,z,t)$, write down the expressions for,
   I. Material derivative
   II. Local acceleration
   III. Convective acceleration

b) A two dimensional velocity field is given by,

$$V = (0.5 + 0.8x)i + (1.5 - 0.8y)j$$

I. Show that the given flow is steady and incompressible.
II. Determine if there are any stagnation points in this flow field.
III. Calculate the velocity and the acceleration at the point x=2m and y=3m

Q5).

a) By considering a differential fluid element in a flow field, write down the expressions for the surface forces and the body forces.

b) A steady laminar flow of a viscous incompressible fluid between two long parallel plates with spacing ***b*** is driven by the motion of the lower plate moving at a constant velocity ***U*** as shown in the Figure Q5. By applying suitable boundary conditions and assumptions, show that the velocity profile **(u(y))** is given in the form,

$$u(y) = K_1\left(y^2 - by\right) + U\left(1 - \frac{y}{b}\right)$$

Where K_1 is a constants.

c) Find the values of the u(y) for y =0.2cm and y=0.4cm.

[*Take Viscosity =0. 19 Pa.s, U=0.6 m s^{-1}, b=0.5cm and dp/dx=-0.2 Nm^{-3}*]

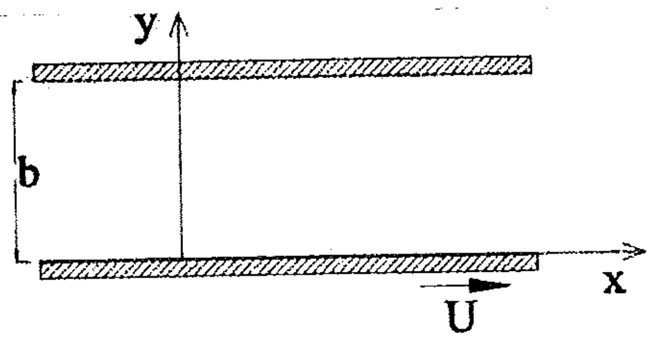

Figure Q5

Q6).

a) State the Buckingham's π theorem.

b) The time period T of surface waves of a fluid is known to depend on the wave length λ, depth of flow D, density of the fluid ρ, acceleration due to gravity g, and surface tension σ. Show that the dimensionless form of the functional relationship can be given as,

$$T\sqrt{\frac{g}{\lambda}} = \phi\left(\frac{D}{\lambda}, \frac{\sigma}{\lambda^2 g\rho}\right)$$

c) What do you understand by dynamic similarity?

d) Oil of density *920 kgm*$^{-3}$ and dynamic viscosity *0.3 Pa s* flows in a pipe of diameter 20 cm at a velocity of 2.4 ms^{-1}. What should be the flow velocity of water in a pipe of diameter 15 cm, to make the two flows dynamically similar? [Take the viscosity of water *0.0013 Pa s*]

Q7).

a) What do you understand by non-Newtonian fluid flows? Explain by drawing graphs of strain rate vs shear stress.

b) The velocity distribution of a viscous liquid flowing over a fixed plate is given by $u_x=2y-y^2$ (u_x is velocity in ms^{-1} and y is the distance from the plate in m). Determine the velocity gradient and shear stress at the boundary and 0.15 m from it. [Viscosity of fluid = $0.9\ N\,s\,m^{-2}$]

Q8).

The equation of the forced vortex flow in usual notation is given by,

$$dp = \frac{\rho\omega^2 r^2}{r} dr - \rho g dz$$

a) Show that the height difference Z_k as shown in Figure Q7 can be written as,

$$Z_k = \frac{\omega^2 r^2}{2g}$$

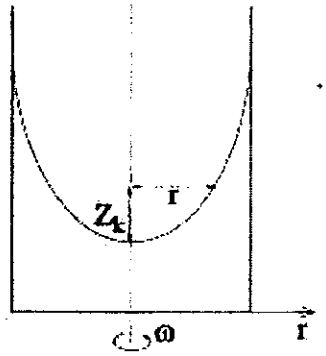

Figure Q7

b) An open circular cylinder of diameter 15 cm and height 100 cm contains water up to a height of 80 cm. Find the maximum speed at which the cylinder is to be rotated about its vertical axis so that no spillage occurs.

Navier-Stokes equations for incompressible flow:-

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

Gradient operator in cylindrical coordinates $\nabla = \left(\frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{\theta} + \frac{\partial}{\partial z}\hat{z}\right)$

------------------------ *End of Paper* ------------------------