The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Civil Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: CVX 5531 Mechanics of Fluids**
Academic Year	: 2020/21
Date	: 30th January 2022
Time	: 0930-1230hrs
Duration	**: 03 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **EIGHT (08)** questions on **FIVE (05)** pages.
3. Answer any **FIVE (05)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Necessary additional information is provided.
6. This is a Closed Book Test (CBT).
7. Answers should be in clear hand writing.
8. Do not use Red colour pen.
9. Take,

   Density of water = 1000 kgm^{-3} Acceleration due to gravity = 9.81 ms^{-2}

   Kinematic viscosity of water = 8.36×10^{-5} m^2/s at 28 °C

Question 01

(a) The velocities in a three-dimensional flow field in x-direction and y-direction are given by,

$u = xy^2$ and $v = -2yz^2$.

(i) Obtain the expression for velocity in z-direction, w if $w(1,0,0) = 1$.

(ii) Determine the acceleration at the point (1,2,1).

(12 marks)

(b) The velocity components in a two-dimensional flow are specified by,

$u = y^3 + 6x - 3x^2y$ and $v = 3xy^2 - 6y - x^3$.

(i) Obtain the velocity potential function demonstrating the flow is irrotational.

(ii) Obtain the complementary stream function.

(08 marks)

Question 02

(a) A hydraulic jump is formed in a horizontal section of an open channel. Neglecting weight component and friction forces and in the absence of external forces,

(i) Prove that the initial and sequent water depths (y_1 and y_2) of the hydraulic jump are related by,

$$y_2 = 0.5y_1\left[\left(1 + 8Fr_1^2\right)^{1/2} - 1\right]$$

where Fr_1 is the Froude number at the initial water depth of the jump.

(ii) Obtain the expressions for non-dimensional initial and sequent depths (i.e. y_1 / E_1 and y_2 / E_1) in terms of Fr_1 and graphically demonstrate their variation with Fr_1. E_1 is the specific energy at the initial water depth.

(12 marks)

(b) A wide channel of uniform rectangular section with a slope of 1/100 has a flow rate of 3.85 m^3/s/m. The Manning's coefficient is estimated to be 0.012 for the channel. If the slope changes suddenly to 1/1600,

(i) Show that a hydraulic jump has to occur.

(ii) Determine the downstream water depth.

(08 marks)

The Manning's equation is given by, $v = \frac{1}{n} R^{2/3} S_0^{1/2}$

Question 03

When a uniform flow of velocity, U flows over a source-sink pair of equal strength, q, the resultant flow is similar to the flow past a Rankine oval body (Figure Q3).

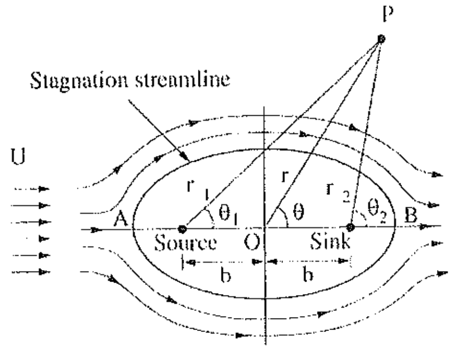

Figure Q3

(a) Show that the locations of the stagnation points, A and B, along the x-axis are given by,

$$x_s = \pm b\sqrt{\left(1+\frac{q}{\pi b U}\right)}$$

(07 marks)

(b) Show that the width, W of the Rankine oval body is given by,

$$W = 2b\cot(\pi U W / 2q)$$

(08 marks)

(c) A bridge pier having the shape of a Rankine oval of length 2.15 m and width 1.34 m is constructed in the middle of a stream. If the stream flows with a uniform velocity 6 m/s, obtain the strength of the source-sink pair considered in the computation of pressure forces acting on the pier.

(05 marks)

The stream function and the velocity potential function for the resultant flow is given by,

$$\psi = Ur\sin\theta + \frac{q}{2\pi}(\theta_1 - \theta_2),\ \phi = Ur\cos\theta + \frac{q}{2\pi}\ln(r_1/r_2)$$

In polar coordinates,

$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta},\ u_\theta = -\frac{\partial\psi}{\partial r}$$

Question 04

(a) Explain the effect of pressure gradient on boundary layer separation.

(07 marks)

(b) Prandtl's boundary layer equation is given by,

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\frac{\partial^2 u}{\partial y^2}$$

where, u and v are the velocities in x and y directions, respectively. The fluid density and kinematic viscosity are denoted by ρ and υ and the pressure is denoted by p.

(i) Using the continuity equation and the momentum equation with no pressure gradient, show that at $y=0$, $\frac{\partial^3 u}{\partial y^3}=0$.

(ii) Check whether the following velocity profile is appropriate for the boundary layer in a flow over a flat plate with no pressure gradient.

$$\frac{u}{U_S}=\frac{3}{2}\frac{y}{\delta}-\frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

U_S is the free stream velocity.

(iii) Obtain the dimensionless boundary layer thickness in a flow over a flat plate, assuming a parabolic velocity profile in the laminar boundary layer.

(13 marks)

Momentum thickness of a boundary layer is given by: $\theta = \int_0^\delta \frac{u}{U_s}\left(1-\frac{u}{U_s}\right)dy$

Shear stress on the solid surface of a boundary layer is given by: $\tau_0 = \rho U_s^2 \frac{d\theta}{dx}$

Question 05

(a) Show that the velocity (V) - time (t) relationship for a horizontal pipeline from a large reservoir resulting from *opening a valve* at the end of the pipeline is given by,

$$t = \frac{L}{(1+K)V_0} \ln\left[\frac{V_0 + V}{V_0 - V}\right]$$

where, V_0 is the final velocity along the pipe after an infinite time, L is the length of the pipe, and K is the ratio of the total head loss in the pipe to velocity head. Assume that the pipe is rigid, and the water is incompressible.

(15 marks)

(b) A valve positioned at the discharge end of a 50 m long pipe of diameter 75 mm is opened at a time when the level in the large tank supplying the pipe is 7.5 m above the pipe inlet. Calculate the time taken to attain a velocity 60% of its final value along the pipe. Assume that the total head loss is only due to friction and the friction factor for the pipe, $f = 0.01$.

(05 marks)

Friction head loss along a pipe, $h_f = 4f \frac{L}{d}\frac{V^2}{2g}$

Question 06

(a) Briefly explain the significance of the Reynolds number in pipe flow.

(04 marks)

(b) To study the pressure drop, ΔP in flow of water through a pipe, a model of scale 1/20 is to be used.

(i) Using Buckingham's Pi theorem, obtain the following correlation,

$$\Delta P = \rho V^2 \phi(\mu / \rho V d)$$

where ρ and μ are the density and the dynamic viscosity of water, respectively. V is the flow velocity, and d is the diameter of the pipe.

(ii) If the pressure drop measured in the model is 1.0 kN/m^2, determine the corresponding pressure drop in the prototype.

(iii) Now, if air of density 1.17 kg/m^3 and dynamic viscosity 1.81 x 10^{-5} Ns/m^2 is used in the model, what is the ratio of pressure drop between the model and the prototype. Take the temperature as 28 ^{0}C.

(16 marks)

Question 07

(a) Differentiate between reaction and impulse turbines

(06 marks)

(b) A Pelton wheel turbine has two jets striking the wheel. The head and the total flow rate available are 320 m and 2.25 m^3/s. The penstock pipe from the reservoir to the turbine is 3000 m long. The efficiency of the power transmission through the pipeline and the nozzle is 92%, and the hydraulic efficiency of the turbine is 89%. The coefficient of velocity of the nozzle is 0.97. If the friction factor, f for the pipe is 0.005, determine the,

(i) Pipe diameter

(ii) Power developed

(14 marks)

Friction head loss along a pipe, $h_f = 4f\frac{L}{d}\frac{V^2}{2g}$

Question 08

(a) Explain the operation of pumps installed in series and in parallel.

(05 marks)

(b) What is meant by the specific speed of a pump, and how are pumps classified according to specific speeds?

(04 marks)

(c) A centrifugal pump running at 1200 rpm develops a head of 24 m while delivering 0.24 m^3/s. The vane angle (i.e. angle vane makes with the tangential direction) at the outlet is 34^0, and the manometric efficiency of the pump is 81%. If the flow velocity is 3.1 m/s, determine the width of the impeller. First, obtain an expression for the theoretical head provided by the pump.

.(11 marks)