The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Civil Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: CVX 5242 Mechanics of Fluids**
Academic Year	: 2020/21
Date	: 30th January 2022
Time	: 0930-1230hrs
Duration	**: 03 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **FIVE (05)** questions on **FOUR (04)** pages.
3. Answer **ALL FIVE (05)** questions. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Necessary additional information is provided.
6. This is a Closed Book Test (CBT).
7. Answers should be in clear hand writing.
8. Do not use Red colour pen.
9. Take,

   Density of water = 1000 kgm^{-3} Acceleration due to gravity = 9.81 ms^{-2}

   Kinematic viscosity of water = 8.36×10^{-5} m^2/s at 28 °C

Question 01

(a) The velocities in a three-dimensional flow field in x-direction and y-direction are given by,

$u = xy^2$ and $v = -2yz^2$.

(i) Obtain the expression for velocity in z-direction, w if $w(1,0,0) = 1$.

(ii) Determine the acceleration at the point $(1,2,1)$.

(12 marks)

(b) The velocity components in a two-dimensional flow are specified by,

$u = y^3 + 6x - 3x^2 y$ and $v = 3xy^2 - 6y - x^3$.

(i) Obtain the velocity potential function demonstrating the flow is irrotational.

(ii) Obtain the complementary stream function.

(08 marks)

Question 02

(a) Show that the velocity (V) - time (t) relationship for a horizontal pipeline from a large reservoir resulting from *opening a valve* at the end of the pipeline is given by,

$$t = \frac{L}{(1+K)V_0} \ln\left[\frac{V_0 + V}{V_0 - V}\right]$$

where, V_0 is the final velocity along the pipe after an infinite time, L is the length of the pipe, and K is the ratio of the total head loss in the pipe to velocity head. Assume that the pipe is rigid, and the water is incompressible.

(15 marks)

(b) A valve positioned at the discharge end of a 50 m long pipe of diameter 75 mm is opened at a time when the level in the large tank supplying the pipe is 7.5 m above the pipe inlet. Calculate the time taken to attain a velocity 60% of its final value along the pipe. Assume that the total head loss is only due to friction and the friction factor for the pipe, $f = 0.01$.

(05 marks)

Friction head loss along a pipe, $h_f = 4f \dfrac{L}{d} \dfrac{V^2}{2g}$

Question 03

(a) Explain the effect of pressure gradient on boundary layer separation.

(07 marks)

(b) Prandtl's boundary layer equation is given by,

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\frac{\partial^2 u}{\partial y^2}$$

where, u and v are the velocities in x and y directions, respectively. The fluid density and kinematic viscosity are denoted by ρ and υ and the pressure is denoted by p.

(i) Using the continuity equation and the momentum equation with no pressure gradient, show that at $y=0$, $\frac{\partial^3 u}{\partial y^3}=0$.

(ii) Check whether the following velocity profile is appropriate for the boundary layer in a flow over a flat plate with no pressure gradient.

$$\frac{u}{U_S}=\frac{3}{2}\frac{y}{\delta}-\frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

U_S is the free stream velocity.

(iii) Obtain the dimensionless boundary layer thickness in a flow over a flat plate, assuming a parabolic velocity profile in the laminar boundary layer.

(13 marks)

Momentum thickness of a boundary layer is given by: $\theta=\int_0^\delta \frac{u}{U_s}\left(1-\frac{u}{U_s}\right)dy$

Shear stress on the solid surface of a boundary layer is given by: $\tau_0=\rho U_s^2\frac{d\theta}{dx}$

Question 04

When a uniform flow of velocity, U flows over a source-sink pair of equal strength, q, the resultant flow is similar to the flow past a Rankine oval body (Figure Q4).

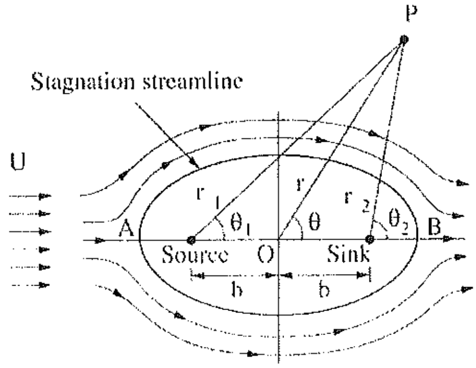

Figure Q4

(a) Show that the locations of the stagnation points, A and B, along the x-axis are given by,

$$x_s = \pm b\sqrt{\left(1+\frac{q}{\pi b U}\right)}$$

(07 marks)

(b) Show that the width, W of the Rankine oval body is given by,

$$W = 2b\cot(\pi U W/2q)$$

(08 marks)

(c) A bridge pier having the shape of a Rankine oval of length 2.15 m and width 1.34 m is constructed in the middle of a stream. If the stream flows with a uniform velocity 6 m/s, obtain the strength of the source-sink pair considered in the computation of pressure forces acting on the pier.

(05 marks)

The stream function and the velocity potential function for the resultant flow is given by,

$$\psi = Ur\sin\theta + \frac{q}{2\pi}(\theta_1 - \theta_2),\ \phi = Ur\cos\theta + \frac{q}{2\pi}\ln(r_1/r_2)$$

In polar coordinates,

$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta},\ u_\theta = -\frac{\partial\psi}{\partial r}$$

Question 05

(a) Consider a compressible fluid filled in a large vessel to which a small nozzle is fitted at the side of the tank. If the pressure drop of the compressible fluid flowing through the nozzle from the vessel is significant (the process is considered to be adiabatic), show that the mass flow rate of the compressible fluid through the nozzle is given by,

$$\dot{m} = \rho_2 A_2 \sqrt{\left(\frac{2\gamma}{\gamma - 1}\right)\frac{p_1}{\rho_1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

where, p_1 and ρ_1 are the pressure and the density of the fluid inside the vessel, respectively. p_2 and ρ_2 are the pressure and the density of the fluid at the exit point. γ is the ratio of the specific heat at constant pressure to the specific heat at constant volume.

(10 marks)

(b) Air at 350 °C flows from a large tank through a converging nozzle of 50 mm diameter. The tank contains air at 160 kN/m^2 and the discharge is to atmosphere of pressure 95 kN/m^2. Calculate the mass flow rate through the nozzle. Take, γ = 1.4 and the universal gas constant, $R = 287$ J/kg.K.

(10 marks)

The Bernoulli's equation for compressible flow undergoing adiabatic process is given by,

$$\left(\frac{\gamma}{\gamma - 1}\right)\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$