The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Civil Engineering

086

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: CVX5533 Structural Analysis**
Academic Year	: 2020/21
Date	: 27th January 2022
Time	: 0930-1230hrs
Duration	**: 3 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Eight (8)** questions in **Five (5)** pages.
3. Answer any **Five (5)** questions only. All questions carry equal marks.
5. Answer for each question should commence from a new page.
6. This is Closed Book Test (CBT).
7. Answers should be in clear hand writing.
8. Do not use Red colour pen.

QUESTION 1

(i) Briefly describe "**how stress is not a vector**" with neat sketches (3 Marks)

(ii) Strain tensor for a homogenous, isotropic material is given below.

$$\sigma_{ij} = \begin{vmatrix} -100 & 0 & -80 \\ 0 & 20 & 0 \\ -80 & 0 & 20 \end{vmatrix}$$

(a) Write six independent stress components (3 Marks)

(b) Determine three stress invariants (4 Marks)

(c) Determine three principal stresses (5 Marks)

(d) Determine three principal strains if the elastic modulus and Poison's ratio are 200 GPa and 0.3, respectively. (5 Marks)

QUESTION 2

(i) Write three stress components according to the Airy's stress function with usual notation. (3 Marks)

(ii) A mathematical function is given below.

$$\emptyset = Ay^3 + Bxy + Cxy^3$$

where A, B and C are constants.

(a) Show that ϕ is an admissible stress function for a cantilever beam with a concentrated load of **P** at the end. The beam has cross section dimensions of width "**b**" and depth "**d**", respectively. The beam length is "**L**". (4 Marks)

(b) State the boundary conditions. (6 Marks)

(c) Determine three stress components. (4 Marks)

(d) Briefly explain the limitations of the above stress function. (3 Marks)

QUESTION 3

(i) Explain three characteristics of statically indeterminate structures (4 Marks)

(ii) A continuous beam (ABCD) is shown in Figure Q3. Flexural rigidities of members AB and CD are equal to 2EI and member BC is 4EI. Uniformly distributed loads (*W*), (*2W*) are acting on members, AB and BC, respectively. There is a concentrated load (*2Wl)* in member CD.

a) Determine the degree of the statical indeterminacy of the beam. (3 Marks)

b) Draw a released structure. (3 Marks)

c) Determine the flexibility matrix for the drawn released structure. (4 Marks)

d) Determine bending moments at B and C. (6 Marks)

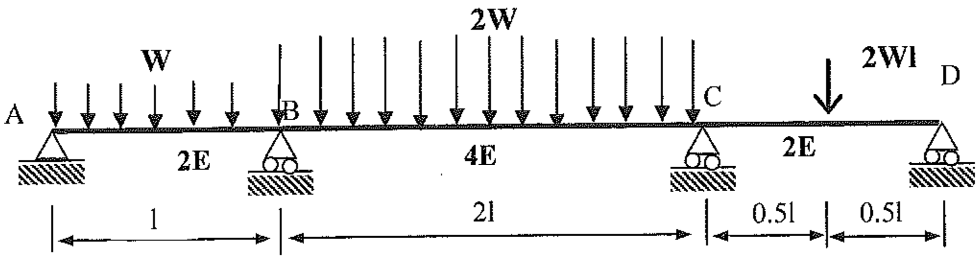


Figure Q3

QUESTION 4

(i) A portal frame structure shown in Figure Q4. All members have same flexural rigidity (EI). You can neglect the axial deformation.

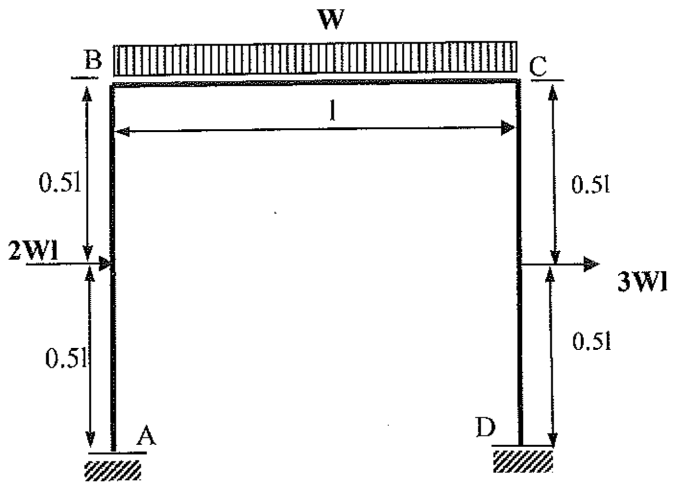


Figure Q4

(i) Find Kinematic indeterminacy of the structure (2 Marks)

(ii) Draw the structure with independent nodal displacements (2 Marks)

(iii) Determine the stiffness matrix of the structures. (4 Marks)

(iv) Find the free nodal displacements at B using the displacement method. (6 Marks)

(v) Using above results, determine the bending moment at B. (6 Marks)

QUESTION 5

(i) List three methods of experimental stress analysis methods.

(2 Marks)

(ii) Briefly explain the difference between **null method** and **out of balance method** in experimental stress measurements using electrical resistance strain gauges. (3 marks)

(iii) The stress state of a certain steel components was determined using a strain rosette as shown in Figure Q5. Due to the loadings, strain gauges gave strain values as $\epsilon_a = 30\ \mu\epsilon$, $\epsilon_b = 50\ \mu\epsilon$, $\epsilon_c = 90\ \mu\epsilon$.

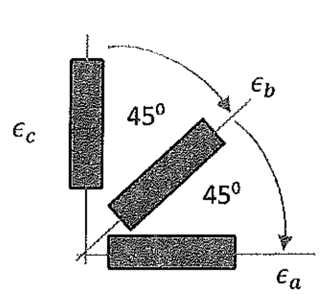


Figure Q5

(a) Draw the Mohr's circle for strains (4 marks)

(b) Determine the in-plane principal strains. (5 marks)

(c) Determine the principal stress, if the bracket material is steel (E = 200GPa, ν = 0.3) (6 marks)

QUESTION 6

(i) Briefly describe three conditions used in plastic analysis of structures. (2 Marks)

(ii) A two-bay frame structure is shown in Figure Q6. Dimensions and plastic moments of the columns and beam are given in the figure.

(a) Draw possible locations of plastic hinge formations. (2 Marks)

(b) Draw elementary failure mechanisms. (2 Marks)

(c) Determine load factors for each elementary failure mechanism. (6 Marks)

(d) Determine the most probable failure mechanism by combining elementary failure mechanisms.

(6 Marks)

(e) Explain how you can ensure the unique solution. (2 Marks)

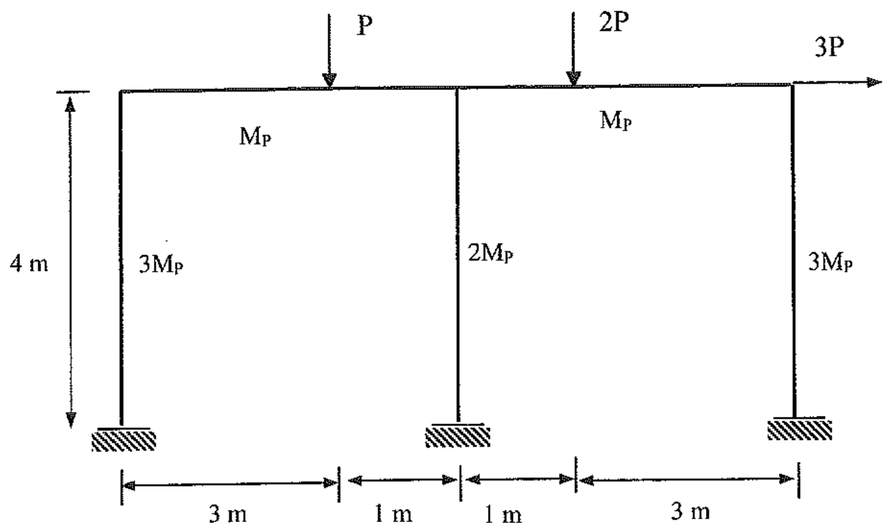


Figure Q6

QUESTION 7

Write short notes on following topics

(i) The difference between static and dynamic loadings and how their applications in engineering structures. (4 Marks)

(ii) The importance of applications of finite element methods in structural analysis (4 Marks)

(iii) The difference between elastic and plastic neutral axes in plastic analysis of beams (4 Marks)

(iv) Application of membrane theory in structural analysis (4 Marks)

(v) Difference between Kinematic indeterminacy and Statical Indeterminacy (4 Marks)

QUESTION 8

The Governing equation for plate bending is given by the following expressions

$$\frac{\partial^4 \omega}{\partial x^4} + \frac{2\partial^4 \omega}{\partial x^2 \partial y^2} + \frac{\partial^4 \omega}{\partial y^4} = \frac{q}{D}$$

All the terms have their normal meanings.

(i) Write three main assumptions used in deriving the plate bending governing equation. (4 Marks)

(ii) A simply supported rectangular plate (Figure Q8) is subjected to a sinusoidal load $q = q_0 \sin \pi x/a \sin \pi y/b$.

If the deflection of the plate can be expressed as $\omega = C \sin \pi x/_a \sin \pi y/_b$

Show that ω is given by the following expression,

$$\omega = \frac{q_o}{D\pi^4\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2} \sin \pi x/_a \sin \pi y/_b$$ (8 Marks)

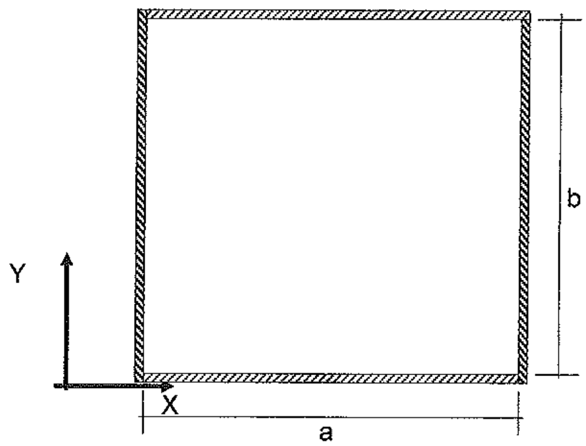


Figure Q8

(iii) The first successful solution for the governing equation for plate bending was proposed by Navier in 1820.

If $q = q_{(x,y)} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn} \sin m\pi x/_a \sin n\pi y/_b$

Show that any particular coefficient ($a_{m'n'}$) is given by the following expression

$$a_{m'n'} = \frac{4}{ab}\int_0^a\int_0^b q_{(x,y)} \sin(\frac{m\pi x}{a})\sin(\frac{n\pi y}{b})dxdy$$ (5 Marks)

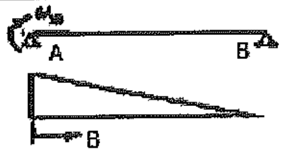
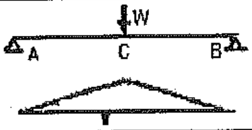
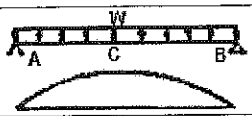
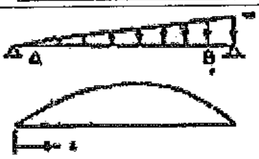
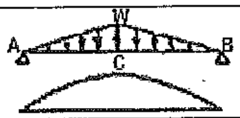
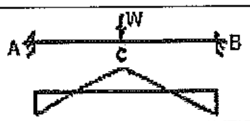
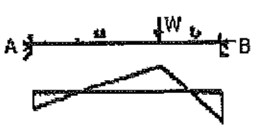
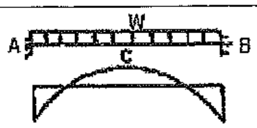
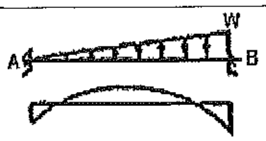
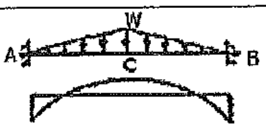
You can assume that,

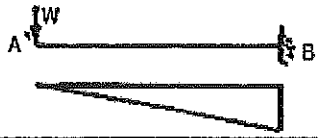
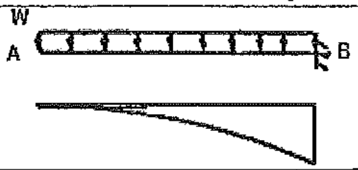
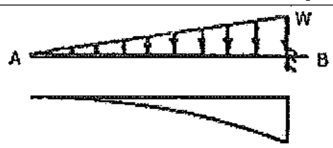
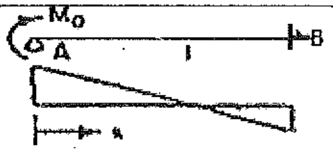
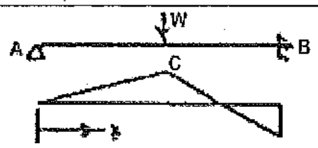
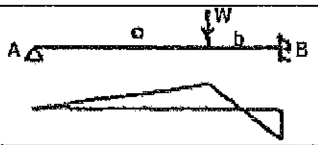
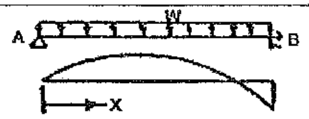
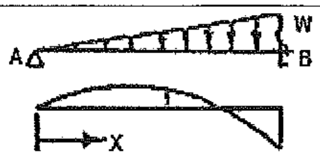
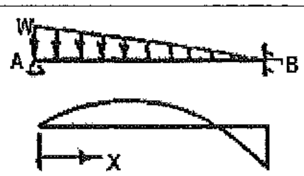
$\int_0^b \sin(\frac{n\pi y}{b})\sin(\frac{n'\pi y}{b})dy = b/2$ when $n=n'$ else $\int_0^b \sin(\frac{n\pi y}{b})\sin(\frac{n'\pi y}{b})dy = 0$

Hence show that for uniformly distributed load

$a_{m'n'} = \frac{4q}{\pi^2 mn}$ when m=1, 3, 5 and n=1, 3, 5

and for even number of m and n $a_{m'n'} = 0$ (3 Marks)

Formulas for Beams				
Structure	**Shear ↿↾**	**Moment ↻↺**	**Stop ∢**	**Deflection ↓**
Simply supported Beam				
	$S_A = -\frac{M_o}{L}$	M_o	$\theta_A = \frac{M_o L}{3EI}$ $\theta_B = \frac{M_o L}{6EI}$	$Y_{max} = 0.062\frac{M_o L^2}{3EI}$ $Y_c = \frac{WL^3}{48EI}$
	$S_A = \frac{W}{2}$	$M_o = \frac{WL}{4}$	$\theta_A = -\theta_B = \frac{WL^2}{16EI}$	$Y_c = \frac{WL^3}{48EI}$
	$S_A = \frac{Wb}{L}$ $S_B = \frac{Wn}{L}$	$M_o = \frac{Wab}{L}$	$\theta_A = \frac{Wab}{6EIL}(L+b)$ θ_B $= -\frac{Wab}{6EIL}(L+a)$	$Y_o = \frac{Wa^2b^2}{3EIL}$
	$S_A = \frac{WL}{2}$	$M_c = \frac{WL^2}{8}$	$\theta_A = -\theta_B = \frac{WL^3}{24EI}$	$Y_c = \frac{5WL^4}{384EI}$
	$S_A = \frac{WL}{6}$ $S_B = \frac{WL}{3}$	M_{max} $= 0.064WI^2$ at $x = 0.577L$	$\theta_A = \frac{7WL^3}{360EI}$ $\theta_B = -\frac{8WL^3}{360EI}$	$Y_{max} = 0.00652\frac{WL^4}{EI}$ at $x = 0.519L$
	$S_A = \frac{WL}{4}$	$M_c = \frac{WL^2}{12}$	$\theta_A = -\theta_B$ $= \frac{5W\ L^3}{192EI}$	$Y_c = \frac{WL^4}{120EI}$
Fixed Beams				
	$S_A = \frac{W}{2}$	$M_c = \frac{WL}{8}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^3}{192EI}$
	$S_A = \frac{Wb^2}{L^3}(3a+b)$ $S_B = \frac{Wa^2}{L^3}(3b+a)$	$M_A = -\frac{Wab^2}{L^2}$ $M_B = -\frac{Wba^2}{L^2}$	$\theta_A = \theta_B = 0$	$Y_o = \frac{Wa^3b^3}{3EIL^3}$
	$S_A = \frac{WL}{2}$	$M_A = M_B$ $= -\frac{WL^2}{12}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^4}{384EI}$
	$S_A = \frac{3WL}{20}$ $S_B = \frac{7WL}{20}$	$M_A = -\frac{WL^2}{30}$	$\theta_A = \theta_B = 0$	$Y_{max} = 0.00131\frac{WL^4}{EI}$ at $x = 0.525L$
	$S_A = \frac{WL}{4}$	$M_A = M_B$ $= -\frac{SWL^2}{96}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{0.7WL^4}{384EI}$

Structure	Shear ↑↾	Moment ↻↺	Stop ∢	Deflection ↓
Cantilever Beam				
	0	M_o	$\theta_A = \frac{M_o L}{EI}$	$Y_A = \frac{M_o L^2}{2EI}$
	W	$M_B = -WL$	$\theta_A = -\frac{WL^2}{2EI}$	$Y_A = \frac{WL^3}{3EI}$
	$S_B = -WL$	$M_B = -\frac{WL^2}{2}$	$\theta_A = \frac{WL^3}{6EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{6}$	$\theta_A = -\frac{WL^3}{24EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{8EI}$	$Y_A = \frac{11WL^4}{120EI}$
Propped Cantilever				
	$S_A = \frac{3M_0}{2L}$	$M_B = \frac{M_0}{2}$	$\theta_A = -\frac{M_0 L}{4EI}$	$Y_{max} = \frac{W_0 L^2}{27EI}$ at $x = \frac{L}{3}$
	$S_A = -\frac{5W}{16}$	$M_B = -\frac{3WL}{16}$ $M_c = -\frac{5WL}{32}$	$\theta_A = -\frac{WL^2}{32EI}$	$Y_{max} = 0.00932\frac{WL^3}{EI}$ at $x = 0.447L$
	$S_A = \frac{Wb^2}{2L^3}(a+2L)$ $S_B = \frac{Wa}{2L^3}(3L^2 - a^2)$	$M_B = -\frac{Wab}{L^2}\left(a + \frac{b}{2}\right)$	$\theta_A = -\frac{Wab^3}{4EIL}$	$Y_0 = \frac{Wa^2 b^3}{12EIL^3}(3L + a)$
	$S_A = \frac{3WL}{8}$	$M_B = -\frac{WL^2}{8}$	$\theta_A = -\frac{WL^3}{48EI}$	$Y_{max} = 0.0054\frac{WL^4}{EI}$ at $x = 0.422L$
	$S_A = \frac{WL}{10}$	$M_{max} = 0.03WL^2$ at $x = 0.447L$ $M_B = -\frac{WL^2}{15}$	$\theta_A = -\frac{WL^3}{120EI}$	$Y_{max} = 0.00239\frac{WL^4}{EI}$ at $x = 0.447L$
	$S_A = \frac{11WL}{40}$	$M_{max} = 0.0423WL^2$ at $x = 0.329L$ $M_B = -\frac{7WL^2}{120}$	$\theta_A = -\frac{WL^3}{80EI}$	$Y_{max} = 0.00305\frac{WL^4}{EI}$ at $x = 0.402L$