The Open University of Sri Lanka
Faculty of Engineering Technology

Study Programme	: Bachelor of Technology (Engineering)
Name of the Examination	: Final Examination
Course Code and Title	: **MEX 6278 / MEX 5230 – FLUID MECHANICS**
Academic Year	: 2015/16
Date	: December 17, 2016
Time	: 9.30 hrs.-12.30 hrs.
Duration	: 3 hours

General Instructions

1. Read all instruction carefully before answering the questions.
2. This question paper consists of 8 questions. All questions carry equal marks.
3. Answer **any 5** questions only.

Q1.

(a). By considering an infinitesimally small cubical fluid element, show that the general equation of continuity in three-dimensional space can be expressed as,

$$\frac{\partial \rho}{\partial t} + \nabla.\left(\rho \vec{V}\right) = 0$$

where ,

$$\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \qquad \text{and} \qquad \vec{V} = u\underline{i} + v\underline{j} + w\underline{k}$$

(b). By using the above relationship, obtain the continuity equation for the following fluid flows.

I. Steady flow
II. Steady - Incompressible flow
III. 2D - Compressible flow

(c). The velocity components for **X** and **Y** directions of a steady incompressible flow are $u = x^2 + y^2 + z^2$ and $v = xy + yz + z$ respectively.
Find the velocity component (w) of the direction **Z** that requires to satisfy the continuity equation.

Q2.

(a). Starting from Nervier-Stokes equations, show that the velocity in steady laminar incompressible flow between two parallel plates, when upper plate moves at a velocity of **U**, in the direction of flow, is given by,

$$\frac{u}{U} = \frac{y}{a} - \frac{1}{2\mu}\frac{dp}{dx}\left(ay - y^2\right)$$

where,
a - distance between plates
μ - viscosity of the fluid
u - flow velocity at a height **y** from the bottom plate

(b). Develop an expression for the total flow rate.

Q3.

(a). Discuss the importance of the Reynolds number in relation to the behaviour of boundary layer.

(b). Explain the significance of the boundary layer displacement thickness $\boldsymbol{\delta^*}$ and the momentum thickness $\boldsymbol{\theta}$. Express $\boldsymbol{\delta^*}$ and $\boldsymbol{\theta}$ as integrals of the boundary layer velocity profiles on a smooth flat plate.

(c). Calculate the ratio $(\boldsymbol{\delta^* / \delta})$ for a laminar boundary layer with a velocity profile given by,

$$\frac{u}{U_\infty} = 2\frac{y}{\delta} - \frac{y^2}{\delta^2}$$

(d). Show that the frictional drag force per unit width (F_d) due to the boundary layer is given by,

$$F_d = \frac{2\delta}{15}\rho U_\infty^{\ 2}$$

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Q4.

(a). By considering the velocity variation that causes rotation and angular deformation in two dimensional differential fluid element given in **Figure Q4**, show that the angular velocity of the fluid flow is given by,

$$\vec{\omega} = \frac{1}{2}\nabla \times \vec{V}$$

where, $\vec{\omega} = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$ and $\vec{V} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$

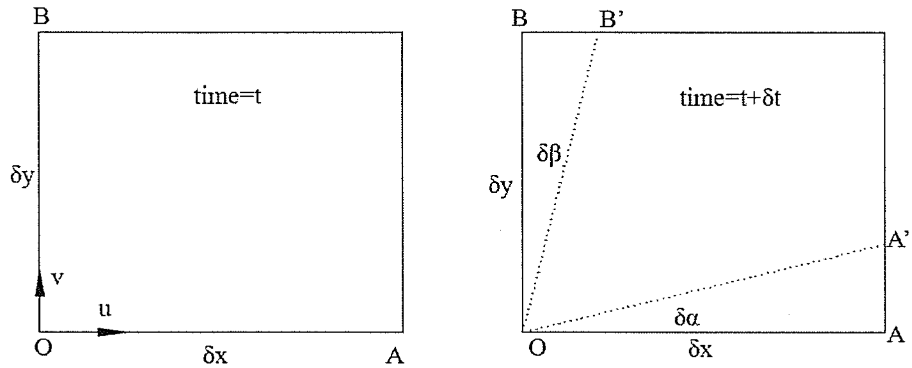

Figure Q4

(b). A velocity field in a plane-flow is given by $V = 2yt\,\underline{i} + x\,\underline{j}$ **ms^{-1}**. Find the acceleration, angular velocity, and the vorticity vector at the point **(4m, 2m)** at **t=3 sec.**

Q5.

(a). What do you understand by potential flow? Briefly explain some applications of potential flow theory.

(b). State the properties of a fluid flow that are required to exist a stream function.

(c). In a two-dimensional incompressible flow, the fluid velocity components are given by: **u = x – 4y** and **v = -y - 4x**. Show that the flow satisfies the continuity equation. Obtain the expression for the stream function. If the flow is potential, obtain the expression for the velocity potential.

Q6.

(a). What do you understand by complex potential and complex velocity?

(b). What is a stagnation point?

(c). Show that the equation of the streamlines due to uniform line sources of strength **Q** through the points **A(-a,0)**, **B(a,0)** and a uniform line sink of strength **2Q** through the origin as depicted in **Figure Q6** is given by,

$$\left(x^2-y^2\right)^2 = 2kxya^2$$

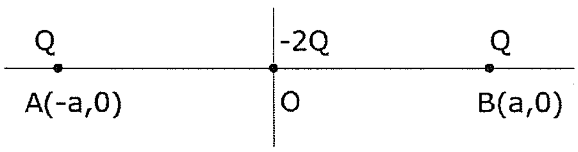

Figure Q6

Q7.

(a). State the assumptions that is required to derive the Reynolds equation.

(b). **Figure Q7** describes the geometry of a step bearing. Simplify the Reynolds equation for this application and show that the pressure variation on ***section -1*** and ***section - 2*** can be expressed as,

$$P_1(x) = 2P_{max}\frac{x}{L}$$

$$P_2(x) = 2P_{max}\left(1-\frac{x}{L}\right)$$

where, P_{max} is the pressure at x=L/2

(c). By applying continuity equation, show that,

$$P_{max} = \frac{3u\eta L\left(h_1-h_2\right)}{\left(h_1^{\ 3}+h_2^{\ 3}\right)}$$

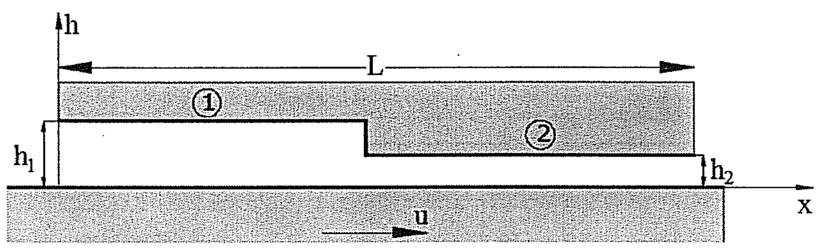

Figure Q7

Q8.

(a). By using suitable examples, explain dimensional homogeneity, geometric similarity, kinematic similarity and dynamic similarity.

(b). State Buckingham π theorem.

(c). By using Buckingham π theorem, show that the frictional torque **T** of a disc of diameter **D** rotating at a speed **N** in a fluid of viscosity **μ** and density **ρ** in a turbulent flow is given by,

$$T = D^5 N^2 \rho \phi\left(\frac{\mu}{D^2 N \rho}\right)$$

(d). A disk of **90mm** in diameter rotates in water having **ρ=1000kg/m³** and **μ=10⁻³ Ns/m²** requires a torque of **0.77 J.** Find the torque required for a similar disc of **240mm** diameter rotating at **3000 rev/min** in air having **ρ=1.2 kg/m³** and **μ=1.8x10⁻⁵ Ns/m²**.

Nervier-Stokes equations for incompressible flow:-

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\rho g_x-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\rho g_y-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=\rho g_z-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

Frictional drag force per unit width due to laminar boundary layer:- $F_d = \rho\int_0^\delta u(U_\infty - u)dy$

Reynolds equation:-

$$\frac{\partial}{\partial x}\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial y}\right)=\frac{(u_h+u_0)}{2}\frac{\partial(\rho h)}{\partial x}+\frac{(v_h+v_0)}{2}\frac{\partial(\rho h)}{\partial y}+\frac{\partial(\rho h)}{\partial t}$$

Acceleration vector:- $\vec{a}=\frac{d\vec{V}}{dt}=\frac{\partial\vec{V}}{\partial t}+u\frac{\partial\vec{V}}{\partial x}+v\frac{\partial\vec{V}}{\partial y}+w\frac{\partial\vec{V}}{\partial z}$

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