00054

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Mechanical Engineering

Study Programme	:	Bachelor of Technology Honours in Engineering
Name of the Examination	:	Final Examination
Course Code and Title	**:**	**MEX5233/DMX5533 Dynamics of Mechanical Systems**
Academic Year	:	2020/21
Date	:	12th February 2022
Time	:	1400 hours -1700 hours
Duration	:	**3 hours**

General instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Eight (08)** questions in **Six (06)** pages.
3. Answer any **Five (05)** questions.
4. Answer for each question should commence from a new page.
5. This is a Closed Book Test (**CBT**).
6. Answers should be in clear handwriting.
7. Do not use Red colour pen.

Question 01 – *(20 Marks)*

Figure Q01 shows a machine of mass m mounted on a foundation through an isolator consisting of a spring of stiffness k and a damper of damping ratio β. The mass is subjected to harmonic force $P \sin \omega t$ where P is the magnitude and ω is the frequency. The motion of the mass is given, in usual notation, as

$$x = \frac{P}{m\omega_n^2} \frac{1}{\sqrt{(1-r^2)^2 + 4\beta^2 r^2}} \sin(\omega t - \phi)$$

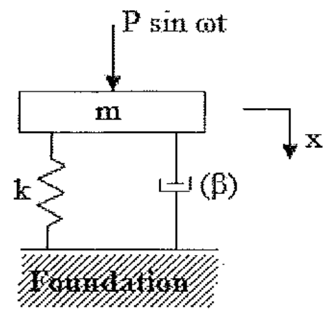

Figure Q01:

(i) Show that the maximum amplitude of motion (X_{max}) is given by $x_{max} = \frac{P}{m\omega_n^2} \frac{1}{2\beta\sqrt{1-\beta^2}}$

and that it occurs at a frequency ratio of $\sqrt{1-2\beta^2}$.

(ii) Derive an expression for the transmissibility ratio of the force transmitted to the foundation.

(iii) When set on a rigid foundation and operating at ***800 rev/min***, a ***1000 kg*** machine tool provides a repeating force of magnitude ***18 kN*** to its foundation. An Engineer has calculated that the maximum repeated force to which the foundation should be subject is ***2600 N***. What is the maximum stiffness of an undamped isolator that provides sufficient isolation between the tool and its foundation?

If the damping ratio of the isolator is ***0.11*** what would be the maximum stiffness of the isolator. If an isolator with these parameters is used what is the steady state amplitude of the machine tool when placed on the isolator and what is the maximum amplitude during start up.

Question 02 - *(20 Marks)*

(a) A spring-mass is excited by a force $F_o \sin \omega t$. At resonance, the amplitude is measured to be ***0.58*** cm. At ***0.80*** resonant frequency, the amplitude is measured to be ***0.46 cm***. Determine the damping factor β of the system.

(b) A machine of ***100 kg*** mass is supported on springs of total stiffness ***700 kN/m*** and has an unbalanced rotating element, which results in a disturbing force of ***350 N*** at a speed of ***3000 rpm***. Assuming a damping factor of $\beta = 0.20$, determine,

(i) its amplitude of motion due to the unbalance,

(ii) the transmissibility, and

(iii) the transmitted force.

Question 03 - *(20 Marks)*

The figure Q 03 shows a steel shaft which can rotate in frictionless bearings, and which has three discs rigidly attached to it in the positions shown. Each of the three discs is solid, uniform, and of diameter ***280 mm***. The modulus of rigidity of the shaft material is ***83 GN/m²***, and the density of the discs is ***7000 kg/m³***. Find the frequencies of free torsional vibrations of the system.

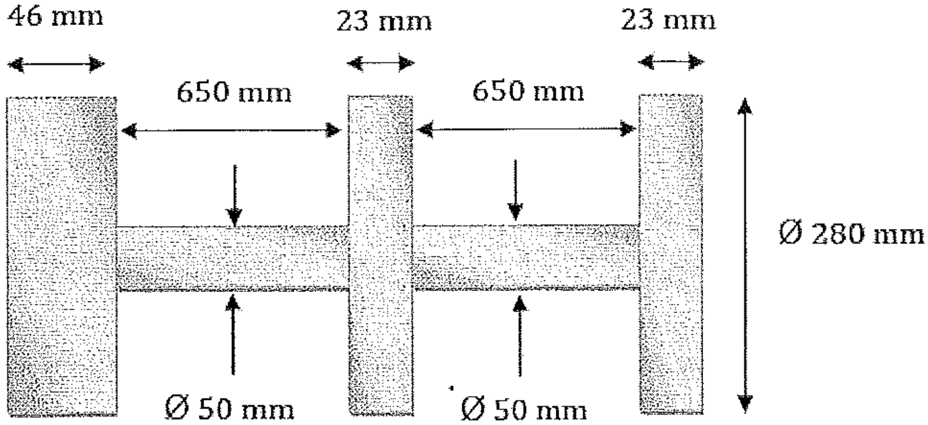

Figure. Q03

Question 04 - *(20 Marks)*

Figure Q04 shows a hydraulic servomotor. A movement in the x direction of the valve is seen to open passage *1* to the supply pressure, which in turn causes the big piston to move to the right. Because the sleeve is directly connected to this piston, the sleeve also moves to the right to close off flow from the valve. Construct the block diagram for this system. Determine the transfer function relating the input position x to the output y. Identify the time constant. Assume that the flow into the big cylinder is proportional to the valve displacement with proportionality constant λ.

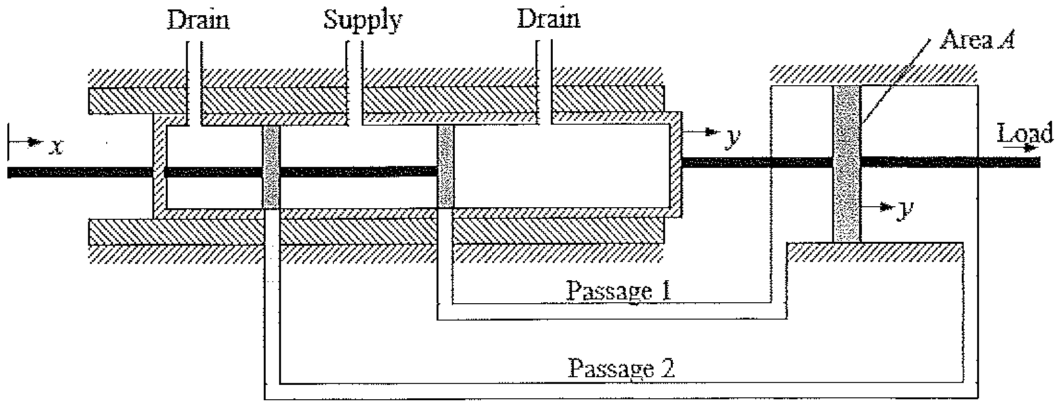

Figure Q04

Question 05 - *(20 Marks)*

Determine the fundamental natural frequency of a fixed-fixed beam using the static deflection curve, $y(x) = \dfrac{\lambda x^2}{24EI}(L-x)^2$, where λ is a constant, L is the length of the beam and EI is the flexural rigidity of the beam.

What would be the natural frequency if the deflection curve takes the form, $y(x) = \lambda\left(1 - \cos\dfrac{2\pi x}{L}\right)$, where λ is a constant and L is the length of the beam.

Question 06 - *(20 Marks)*

(a) Reduce the block diagram shown in Figure Q05 and obtain the transfer function relating the output and input.

(b) Draw the signal flow graph corresponding to the block diagram shown in Figure Q05 and obtain the transfer function relating the output and input using Mason's rule.

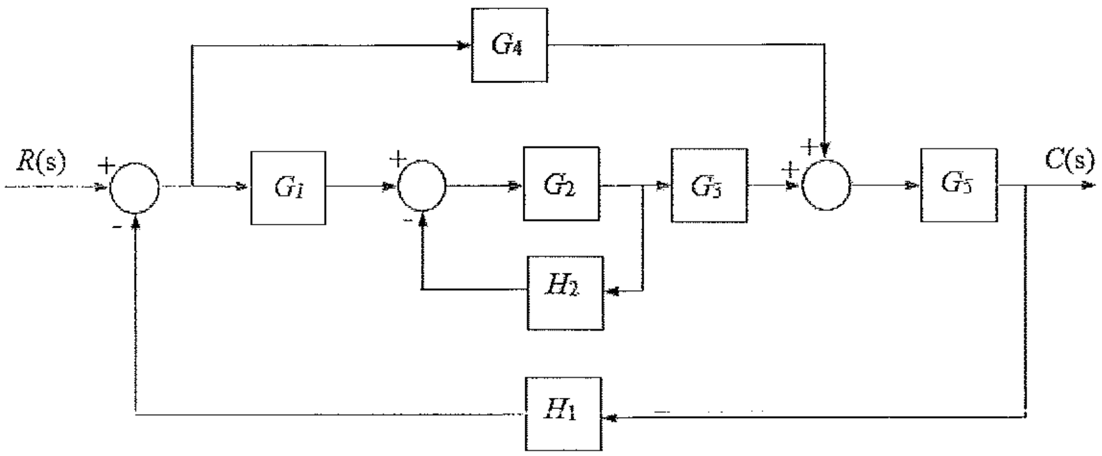

Figure Q05

Question 07 - *(20 Marks)*

The open loop transfer function of a unity feedback control system is given by

$$G(s) = \frac{K}{(s+1)(s+2)(s+5)},$$

where K is the gain of the controller. Draw the root locus as a function of K and determine the range of values of K for which the system is stable.

Using the root locus you have drawn show that s = - 6 is a closed loop pole of the system when $K = 20$ and hence derive an expression for the response of the system for a unit impulse input for $K = 20$. Assume that all initial conditions are zero.

Question 08 - *(20 Marks)*

Figure Q08 shows a feed back control system in which;

$$G(s) = \frac{K}{s^2(s+1)(s+3)},$$

$H(s) = 1$, and K is the gain of the controller. Draw the root locus of the zeros of $1 + G(s)H(s) = 0$ for $0 \leq K < \infty$.

How would the root locus appear if $H(s) = 1 + 5s$.

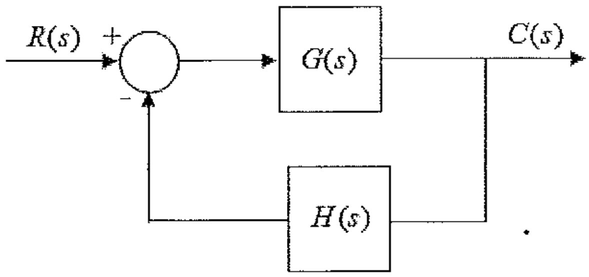

Figure Q08

LAPLACE TRANSFORMS

TIME FUNCTION $f(t)$	LAPLACE TRANSFORM $F(s)$
Unit Impulse $\delta(t)$	1
Unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{df(t)}{dt}$	$sF(s)-f(0)$
$\frac{d^n f(t)}{dt^n}$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} \frac{df(0)}{dt} \ldots - \frac{d^{n-1} f(0)}{dt^{n-1}}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$