The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Electrical & Computer Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: EEX5360- Signals and Systems**
Academic Year	: 2020/2021
Date	: 09th February 2022
Time	: 1400-1700hrs
Duration	**: 3 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of Six **(06)** questions in **five (05)** pages.
3. Answer any **Five (5)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. This is a Closed Book Test (CBT).
6. Answers should be in clear handwriting.
7. **Do not** use a red color pen.
8. Adhere to usual notations.

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1. Figure 1 shows a passive RC circuit with a voltage source.

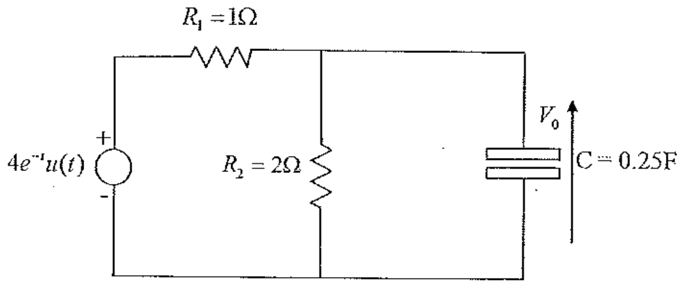

Figure 1: RC Circuit

i. Transform the circuit elements from time domain to the Laplace domain. [08 marks]

ii. Hence, draw the Laplace domain equivalent circuit for the Figure 1. [02 marks]

iii. Find the output voltage (V_0), across the capacitor. [10 marks]

2.

a) A periodic triangular signal is shown in Figure 2. Find its complex Fourier series. [14 Marks]

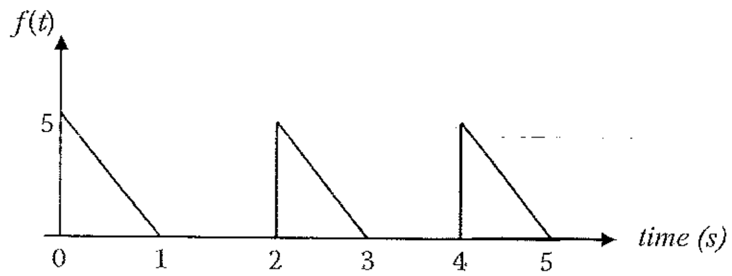

Figure 2: Periodic Triangular wave

b) The input and the impulse response of discrete-time LTI system are given by, $x[n] = (0.9)^n u[n]$ and $h[n] = (0.2)^n u[n]$ respectively. Find the output $y[n]$ of the system. [06 marks]

Hint*: You may use the Geometric series identity $\sum_{K=0}^{n} \left(\frac{a}{b}\right)^k = \frac{1-\left(\frac{a}{b}\right)^{n-1}}{1-\left(\frac{a}{b}\right)}$

3. Figure 3 shows a passive RC circuit with a current source ($I_s = 10e^{-2t}u(t)A$).

   a. Transform the circuit elements from time domain to the frequency domain (Ω - domain). [04 marks]
   b. Find the current through the capacitor using the inverse Fourier transform. [10 marks]

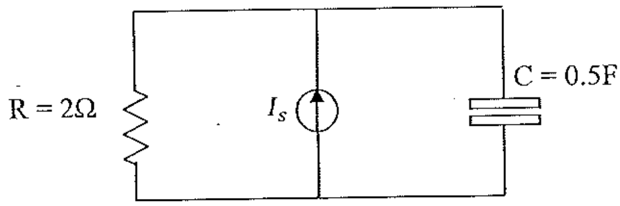

Figure 3: RC Circuit

   c. Find the Fourier Transform of the 'two-sided' exponential pulse shown in Figure 4 and sketch its spectrum. [06 marks]

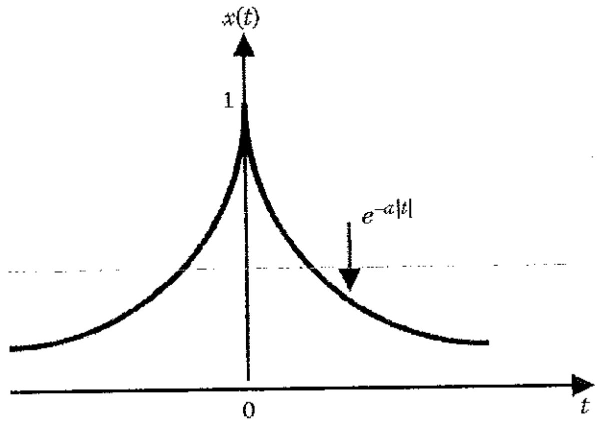

Figure 4: 'Two-sided' exponential Pulse

4.

   a. The continuous-time signal $x(t) = 3\cos(400\pi t)$ is sampled with 600 Hz. Find the digital angular frequency of the corresponding discrete-time signal $x[n]$. [03 marks]

   b. Consider the continuous-time signal $x(t) = 4\cos(100\pi t) + 3\cos^2(200\pi t)$. If $x(t)$ is sampled with a sampling frequency fs Hz, express the condition that should be satisfied by the sampling frequency fs in order to ensure the perfect recovery of $x(t)$ from the corresponding discrete-time signal. [03 marks]

c. Consider the continuous-time signal $x(t) = 2\cos(60\pi t)$.

i. Using the continuous-time Fourier transform pair $cos\Omega_0 t \leftrightarrow \pi[(\Omega - \Omega_0) + (\Omega + \Omega_0)]$, Where Ω_0 is a constant angular frequency in rad/s, sketch the spectrum $X(\Omega)$. [02 marks]

ii. The continuous-time signal $x(t)$ is sampled with 50 Hz in order to generate the discrete-time signal $x[n]$. Sketch the spectrum $X(\omega)$ of $x[n]$ in the range $3\pi \leq \omega \leq 3\pi$. [08 marks]

iii. Assume that $x[n]$ is applied to an ideal reconstruction filter of which the output is $\hat{x}(t)$. What is the frequency (in Hz) of the continuous-time signal $\hat{x}(t)$? [04 marks]

5.

a. Find the z-transform of the following signals with the region of convergence.

(i). $x[n] = a^n u[n]$

(ii). $x[n] = -a^n u[-n-1]$ [05 marks]

Where a is a real-valued constant and $u[n]$ is the discrete-time unit-step function.

b. Using the answer obtained for **Q5-(a)** and the relevant properties of the z-transform or any other means, find the z-transform of the signal with the region of convergence. [05 marks]

$$W[n] = (n-2)a^{n-2}u[n-2]$$

c. Consider an LTI System having the transfer function

$$H(z) = \frac{1}{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{4}z^{-1})} \quad ; |z| > \frac{1}{2}$$

Using the answer obtained for **Q5-(a)** and the relevant properties of the z-transform, find the output signal $y[n]$ of the system, if the input signal is $u[n]$. [10 marks]

6.

i. The coefficients of an N th order (length $N+1$), where N is even) FIR filter H(z) is denoted as $h[n]$, $-N/2 \leq n \leq N/2$. Express the condition that should be satisfied by the coefficients $h[n]$ of the FIR filter in order to have a zero-phase response.

[05marks]

ii. The ideal frequency response of a zero-phase bandpass filter H(z) is specified as

$$H_l(e^{j\omega}) = \begin{cases} 1, & for\ \omega_l \leq |\omega| \leq \omega_u \\ 0, & otherwise, \end{cases}$$

where $-\pi \leq \omega \leq \pi$, and ω_l and ω_u $(0< \omega_l < \omega_u < \pi)$ are the lower and upper cutoff frequencies of the bandpass filter, respectively. Derive a closed-form expression for the infinite-extent ideal impulse response $h_l[n]$. [15 marks]

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