00022

THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Bachelor of Technology Honors in Engineering / Bachelor of Software Engineering Honors

Final Examination (2020/2021)
MHZ5340 / MHZ5360 : Discrete Mathematics II

Date: 30th January 2022 (Sunday) **Time: 14:00 – 17:00**

Instruction:

- Answer only six questions.
- Please answer a total of six questions choosing at least one from each single section.
- All symbols are in standard notation and state any assumption that you made.

SECTION – A

Q1.

i. Let A be any nonempty set with the operation " $*$ " defined by

$x * y = x^2 + y^2 + 2xy$.

a. Is the operation associative?

b. Is the operation commutative?

[30%]

Justify your answer.

ii. Define a semi group $(G, *)$ in usual notation.

Prove weather $(\mathbb{R}, *)$ is a semi group if the operation " $*$ " defined as follows.

[35%]

a. $a * b = a + 3b - 1$

b. $x * y = \sqrt{xy}$

iii. Let $(R, \oplus_5)$ be a group where $R = \{0, 1, 2, 3, 4\}$ and $\oplus_5$. The operation $\oplus_5$ defined by $a \oplus_5 b = r$ and $0 \leq r \leq 4$, where r is the non-negative remainder when ordinary addition $a + b$ is divided by 5.

[35%]

a. Determine the identity element of $\mathbb{R}$.

b. Determine the inverse of each element $a \in \mathbb{R}$.

Q2.

i. Define a group $(G, *)$ and an Abelian group with usual notation [10%]

ii. Let $G = \left\{\begin{bmatrix}1 & a\\0 & 1\end{bmatrix} \middle| \ a \in \mathbb{Z}\right\}$ and " $*$ " denote the usual matrix multiplication.

a. Show that $(G,*)$ is a group.

b. Determine whether $(G,*)$is an Abelian group or not [50%]

iii. Let $S = \mathbb{R}\backslash\{-1\}$ and a binary operation " $*$ " is defined on S by

$$a * b = a + b + ab$$

Prove that $(S,*)$ is an Abelian group. [40%]

Q3.

i. Define a homomorphism and Isomorphism for group in usual notation. [20%]

ii. Let $G = (R, +)$ and $G_1 = (R^1, \cdot)$. Also $\phi: G \longrightarrow G_1$, defined by $\phi(x) = 10^x$. Show that ϕ is a homomorphism. [30%]

iii. Let $G = \left\{\begin{bmatrix}a & b\\c & d\end{bmatrix} \middle| \ a, b, c, d \in \mathbb{R}\right\}$, and $G' = \mathbb{R}$. Assume that G and G' are groups under the usual matrix multiplication. Let $\phi: G \longrightarrow G'$, defined by $\phi(A) = ad - bc$, where $A = \begin{bmatrix}a & b\\c & d\end{bmatrix} \in G$.

Show that ϕ is an Isomorphism. [50%]

SECTION – B

Q4.

i. Define a simple graph. [15%]

ii. By drawing each of the following graph, determine which of those graphs are simple or not. [45%]

a. $G_1 = \{V_1, E_1\}$ where $V_1 = \{1, 2, 3, 4, 5, 6\}$ and
$E_1 = \{\{x, y\} \mid (x + y) \text{ is even and } x \leq y\}$

b. $G_2 = \{V_2, E_2\}$ where $V_2 = \{1, 2, 3, 4, 5, 6,7\}$ and
$E_2 = \{\{x, y\} | 1 < \frac{x}{y} < 2\}$

c. $G_3 = \{V_3, E_3\}$ where $V_3 = \{1, 2, 3, 4, 5, 6\}$ and
$E_3 = \{\{1,2\}, \{1,3\}, \{1,4\}, \{2,5\}, \{2,6\}, \{3,6\}, \{4,5\}\}$

iii. Let G be a graph of 8 vertices and 15 edges such that every vertex is of degree 3 or 5. How many vertices of G are degree 3 and 5. Construct one such graph G. [40%]

Q5.

i. Define a tree graph and the forest. [10%]

ii. Is it possible to draw a tree for the each of the following cases? [40%]

a. 12 vertices, each of which has either degree 1 or degree 3.

b. 16 vertices, exactly 12 of which have degree of one.

iii. Six kids, Joe, Kay, Jim, Bob, Rae and Kim, play a variation of hide and seek. The hiding place of a child is known only to a select few of the children. A child is then paired with another with the objective of finding the partner's hiding place. This may be achieved through a chain of other kids who eventually will lead to discovering where the designated child is hiding. For example, suppose that Joe needs to find others and Joe knows where Jim is hiding, who in turn knows where Kim. Thus, Joe can find Kim by first finding Jim, who in turn will lead Joe to Kim. The following list provides details of the children: [50%]

- Joe knows the hiding place of Bob and Kim
- Kay knows the hiding place of Bob, Jim and Rae
- Jim knows the hiding place of Kay
- Bob knows where Kay, Joe and Kim are hiding
- Ray knows where Kay and Kim are hiding
- Kim knows where Rae, Joe and Bob are hiding

a. Draw the graph representing the above relationship.

b. Find the degree for each vertex of the graph.

c. Write down the adjacency matrix for the graph. [50%]

Q6.

i. Define the connected graph. [10%]

ii. G is the graph whose adjacency matrix A is given by, [50%]

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}.$$

a) Without drawing a graph of G, explain whether G is connected or not.

b) If $V(G) = \{a, b, c, d\}$ then find the number of paths of length four joining vertices d and b.

c) Draw the graph of an adjacency matrix A.

iii. What is the largest possible number of vertices in a graph with 8 edges if all the vertices have degree at least four? [40%]

SECTION – C

Q7.

i. Iterate the Eco-system growth for the relation $y_{n+1} - \lambda y_n = 0$ for $y_0 = 0.5$, taking $\lambda = 0.4$ and $\lambda = 1.4$ (at least 5 iteration steps are required) and draw the diagram. Hence deduce k_n as $n \to \infty$. **[50%]**

ii. Iterate the Eco-system growth model relationship $x_{n+1} = \lambda x_n(1 - x_n)$, where $\lambda = 2$ and $y_0 = 0.6$ (at least 5 iteration steps are required) and draw the diagram. Hence deduce k_n as $n \to \infty$. **[25%]**

iii. Iterate the relationship $x_{n+1} = \lambda x_n(1 - x_n)$,where $\lambda = 2$ and $x_0 = 0.7$ and state whether the relation is attractor period one or attracter period two. **[25%]**

Q8. A three-dimensional system is governed by the following system of differential equations:

$$\frac{dx}{dt} = 3x - y - z$$

$$\frac{dy}{dt} = -x + 3y + z$$

$$\frac{dz}{dt} = x + y + 3z$$

where x, y and z are function of t and at $t = 0, (x, y, z) = (1, 0, 1)$.
Find the phase space value $(x,\ y,\ z)$ at $t = 1, 2$. **[100%]**

Q9.

i. Let $L = \{001, 10, 111\}$ and $M = \{\varepsilon, 01, 001\}$ be languages. Find the concatenations LM and ML. **[20%]**

ii. Construct the production rules to generate each of the following language: **[30%]**

a) $\{a^n : n \geq 1\}$.

b) $\{a^{2n} : n \geq 1\}$.

c) $\{(bc)^n : n \geq 1\} \cup \{(cb)^n : n \geq 1\}$.

iii. Show that the string $a - b * c + d$ is a sentence generated by the grammar G, where $G = \{\{S, E, T, F\}, \{+, *, -, /, a, b, c, d\}, P, S\}$ and starting symbol S and production P. [15%]

P is the set of rules defined as follows:

$S \to E,$	$T \to F$
$E \to E + T,$	$F \to (E)$
$E \to E - T,$	$F \to a,$
$E \to T,$	$F \to b,$
$T \to T * F$	$F \to c,$
$T \to T/F$	$F \to d$

III. Let *M* be a finite-state machine with state table given below: [35%]

State	**Input**			**Output**		
	a	*b*	*c*	*a*	*b*	*c*
A	A	C	D	1	1	1
B	A	B	B	0	0	1
C	D	C	B	0	1	0
D	D	C	D	1	1	1

a) Find the input set, the state set, and the output set.
b) Draw the state diagram of *M*.
c) Starting at A, what is the output for the input string $aabbacb$?

--- END ---