

THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics and Philosophy of Engineering
Bachelor of Technology Honors in Engineering



Final Examination (2020/2021)
MHZ5530: Engineering Mathematics III

Date: 24th February 2022

Time: 0930 hours-1230 hours.

Instructions:

- This paper consists of NINE(09) questions in FIVE(05) pages.
- Answer any SIX(06) questions only . All questions carry equal marks.
- Provide answers in separate sheets (answer booklet) which will be given in the examination.
- Show all your workings.
- Answer for each question should commence from a new page.
- State any assumptions you required.
- All the symbols are in standard notation unless they are defined.
- All the relevant statistical tables are attached with this paper.
- This is a closed book test.
- Do not use red color pen.

1. (a) Show that $\text{div}(\text{grad}r^n) = n(n+1)r^{n-2}$, where $r = \sqrt{x^2 + y^2 + z^2}$. Hence show that $\nabla^2\left(\frac{1}{r}\right) = 0$.

(Marks 35)

- (b) Find the value of $\iint_S ((\vec{F} \cdot \vec{n})dA$, using the Gauss Divergence Theorem over the entire surface of the region above the xy -plane bounded by the cone $z^2 = x^2 + y^2$ and the plane $z = 4$, if $\vec{F} = 4xz\mathbf{i} + xyz^2\mathbf{j} + 3z\mathbf{k}$.

(Marks 65)

2. (a) Find the value of the surface integral of the function $f(x, y) = y - x$, throughout the area enclosed by the curves $y - x = 2$, $y + x = 2$ and x axis.

(Marks 65)

- (b) Find the volume of the cylinder $x^2 + y^2 = 2x$, enclosed between the planes $z = 1$ for $z = 0$.

(Marks 35)

3. (a) By the Green's Theorem area A of the region bounded by a closed curve C is given by,

$$\frac{1}{2} \oint_C (xdy - ydx)$$

Hence find the area of the region in the first quadrant bounded by the curves, $y = x$, $y = \frac{1}{x}$ and $y = \frac{x}{4}$.

(Marks 35)

- (b) Verify the Stoke's Theorem for $\vec{F} = (x + y)\mathbf{i} + (2x - z)\mathbf{j} + (y + z)\mathbf{k}$ for the surface of a triangular lamina with vertices $(2, 0, 0)$, $(0, 3, 0)$ and $(0, 0, 6)$.

(Marks 65)

4. (a) i. Define an irrotational vector field.

(Marks 10)

ii. Write down any two properties of an irrotational vector field.

(Marks 10)

- (b) Consider the vector field $\vec{F} = (2xy + z^2)\mathbf{i} + x^2\mathbf{j} + 2xz\mathbf{k}$.

i. Show that the vector field \vec{F} with domain \mathbb{R}^3 is an irrotational vector.

(Marks 20)

ii. Find a scalar potential function ϕ such that $F = \text{grad}\phi$.

(Marks 30)

- (c) Evaluate $\oint_c 3x^2 dx + 2x dy$ using Green's Theorem (in plane), where c is the boundary of the closed region bounded by $y = 1$ and $x = \sqrt{y}$ in counter clockwise direction. (Marks 30)
5. (a) Find the Laurent series expansion for $f(z) = \frac{2z}{z^2 + 4}$ around $2i$. (Marks 30)
- (b) Find the Maclaurin series expansion for the complex valued function $f(z) = e^z$. (Marks 30)
- (c) Show that the image of the half plane $A = \{z : \operatorname{Re}(z) \geq \frac{1}{4}\}$ under mapping $w = \frac{1}{z}$ is the closed disk $D = \{w : |w - 2| \leq 2\}$. (Marks 40)
6. (a) Suppose that there is a rotation of the coordinate system $OXYZ$ through the axis OZ by the angle $\frac{\pi}{3}$ anti-clockwise direction. Consider the vector $\vec{A} = \sqrt{3}\mathbf{i} + \mathbf{j} + \mathbf{k}$ which is in rotated coordinate system, where \mathbf{i} , \mathbf{j} and \mathbf{k} are the unit vectors with respect to the OX' , OY' and OZ' respectively. Then find the vector \vec{A} with respect to the initial coordinate system. (Marks 25)
- (b) The components of the moment of inertia tensor of an object with respect to a system of coordinates is given by,

$$T_{ij} = \begin{bmatrix} 8 & -3 & -3 \\ -3 & 8 & -3 \\ -3 & -3 & 8 \end{bmatrix}$$

- i. Find the Eigen values and Eigen vectors of the tensor T_{ij} . (Marks 30)
- ii. Find the principal components of the tensor T_{ij} . (Marks 15)
- iii. Verify the result in the above part (b) ii, by using the Transformation Law of tensors. (Marks 30)
7. (a) Use the method of separation of variables to solve the following partial differential equation,

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

Given that $u = \sin \pi x + \sin 8\pi x - \sin 20\pi x$, when $t \rightarrow \infty$, and $u = 0$ at $x = 0$ and $x = 1$.

(Marks 50)

(b) Using the Fourier Transform, solve the partial differential equation given by,

$$\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2} \quad \text{for } 0 \leq x < \infty, t > 0$$

for the given conditions,

- $u(x, 0) = 0$ for $x \geq 0$
- $\frac{\partial u}{\partial x}(0, t) = -2$
- $u(x, t)$ is unbounded

(Marks 50)

8. The lifetime of four brands of Flashlight batteries are to be compared by testing each brand in a sample of five flash lights batteries. The lifetimes of the each batteries to the nearest hour are as follows .

Brand A	Brand B	Brand C	Brand D
42	28	24	20
30	36	36	32
39	31	28	38
28	32	28	28
29	27	33	25

Let a group of customers claims that there appear to be a difference in mean lifetime among the four brands of batteries.

- (a) Identify the factor variable and dependent variable. (Marks 10)
- (b) State the null and alternative hypothesis associated with the customers claim. (Marks 10)
- (c) Test the hypothesis about the customers claim at 5% significant level using ANOVA table and interpret the findings. (Marks 70)
- (d) State the assumptions you made for this test. (Marks 10)

9. A firm produces three products. These products are proceeded on three different machines. The time required to manufacture one units of each of three product and the manufacturing capacity of three machines are given in following table.

Machine	Time per unit(Hours)			Machine Capacity (Hours per week)
	Product 1	Product 2	Product 3	
M1	4	1	1	30
M2	2	3	1	60
M3	1	2	3	40

The profit per unit for product 1, product 2 and product 3 are Rs.30/ =, Rs.20/ = and Rs.10/ = respectively. It is assumed that all the products produced are consumed in the market.

- (a) Identify the decision variables. (Marks 10)
- (b) Formulate a mathematical model(Linear Programming Model) to estimate the production structure which will maximize weekly profit. (Marks 25)
- (c) Convert the linear programming problem in part (b) to standard form of the linear programming model. (Marks 15)
- (d) Write down the basic solution of the initial tabular of the simplex method which use to solve the linear programming problem in part (c). (Marks 10)
- (e) Solve the linear programming problem in part (c) using simplex method. (Marks 40)

End.

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F Distribution: Critical Values of F (5% significance level)

v_1	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20
1	161.45	199.50	215.71	234.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.36	246.46	247.32	248.01
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.42	19.43	19.44	19.45
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.71	8.69	8.67	8.66
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.87	5.84	5.82	5.80
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.64	4.60	4.58	4.56
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.96	3.92	3.90	3.87
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.53	3.49	3.47	3.44
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.24	3.20	3.17	3.15
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.03	2.99	2.96	2.94
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.86	2.83	2.80	2.77
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.74	2.70	2.67	2.65
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.64	2.60	2.57	2.54
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.55	2.51	2.48	2.46
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.48	2.44	2.41	2.39
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.42	2.38	2.35	2.33
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.37	2.33	2.30	2.28
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.33	2.29	2.26	2.23
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.29	2.25	2.22	2.19
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.26	2.21	2.18	2.16
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.22	2.18	2.15	2.12
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.20	2.16	2.12	2.10
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.17	2.13	2.10	2.07
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.15	2.11	2.08	2.05
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.13	2.09	2.05	2.03
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.11	2.07	2.04	2.01
26	4.22	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.09	2.05	2.02	1.99
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.08	2.04	2.00	1.97
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.06	2.02	1.99	1.96
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.05	2.01	1.97	1.94
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.04	1.99	1.96	1.93
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.04	1.99	1.94	1.91	1.88
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.95	1.90	1.87	1.84
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.95	1.89	1.85	1.81	1.78
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.86	1.82	1.78	1.75
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97	1.89	1.84	1.79	1.75	1.72
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.88	1.82	1.77	1.73	1.70
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94	1.86	1.80	1.76	1.72	1.69
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.85	1.79	1.75	1.71	1.68
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.78	1.73	1.69	1.66
150	3.90	3.06	2.66	2.43	2.27	2.16	2.07	2.00	1.94	1.89	1.82	1.76	1.71	1.67	1.64
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.80	1.74	1.69	1.66	1.62
250	3.88	3.03	2.64	2.41	2.25	2.13	2.05	1.98	1.92	1.87	1.79	1.73	1.68	1.65	1.61
300	3.87	3.03	2.63	2.40	2.24	2.13	2.04	1.97	1.91	1.86	1.78	1.72	1.68	1.64	1.61
400	3.86	3.02	2.63	2.39	2.24	2.12	2.03	1.96	1.90	1.85	1.78	1.72	1.67	1.63	1.60
500	3.86	3.01	2.62	2.39	2.23	2.12	2.03	1.96	1.90	1.85	1.77	1.71	1.66	1.62	1.59
600	3.86	3.01	2.62	2.39	2.23	2.11	2.02	1.95	1.90	1.85	1.77	1.71	1.66	1.62	1.59
750	3.85	3.01	2.62	2.38	2.23	2.11	2.02	1.95	1.89	1.84	1.77	1.70	1.66	1.62	1.58
1000	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89	1.84	1.76	1.70	1.65	1.61	1.58

F Distribution: Critical Values of F (5% significance level)

v_1	25	30	35	40	50	60	75	100	150	200
v_2										
1	249.26	250.10	250.69	251.14	251.77	252.20	252.62	253.04	253.46	253.68
2	19.46	19.46	19.47	19.47	19.48	19.48	19.48	19.49	19.49	19.49
3	8.63	8.62	8.60	8.59	8.58	8.57	8.56	8.55	8.54	8.54
4	5.77	5.75	5.73	5.72	5.70	5.69	5.68	5.66	5.65	5.65
5	4.52	4.50	4.48	4.46	4.44	4.43	4.42	4.41	4.39	4.39
6	3.83	3.81	3.79	3.77	3.75	3.74	3.73	3.71	3.70	3.69
7	3.40	3.38	3.36	3.34	3.32	3.30	3.29	3.27	3.26	3.25
8	3.11	3.08	3.06	3.04	3.02	3.01	2.99	2.97	2.96	2.95
9	2.89	2.86	2.84	2.83	2.80	2.79	2.77	2.76	2.74	2.73
10	2.73	2.70	2.68	2.66	2.64	2.62	2.60	2.59	2.57	2.56
11	2.60	2.57	2.55	2.53	2.51	2.49	2.47	2.46	2.44	2.43
12	2.50	2.47	2.44	2.43	2.40	2.38	2.37	2.35	2.33	2.32
13	2.41	2.38	2.36	2.34	2.31	2.30	2.28	2.26	2.24	2.23
14	2.34	2.31	2.28	2.27	2.24	2.22	2.21	2.19	2.17	2.16
15	2.28	2.25	2.22	2.20	2.18	2.16	2.14	2.12	2.10	2.10
16	2.23	2.19	2.17	2.15	2.12	2.11	2.09	2.07	2.05	2.04
17	2.18	2.15	2.12	2.10	2.08	2.06	2.04	2.02	2.00	1.99
18	2.14	2.11	2.08	2.06	2.04	2.02	2.00	1.98	1.96	1.95
19	2.11	2.07	2.05	2.03	2.00	1.98	1.96	1.94	1.92	1.91
20	2.07	2.04	2.01	1.99	1.97	1.95	1.93	1.91	1.89	1.88
21	2.05	2.01	1.98	1.96	1.94	1.92	1.90	1.88	1.86	1.84
22	2.02	1.98	1.96	1.94	1.91	1.89	1.87	1.85	1.83	1.82
23	2.00	1.96	1.93	1.91	1.88	1.86	1.84	1.82	1.80	1.79
24	1.97	1.94	1.91	1.89	1.86	1.84	1.82	1.80	1.78	1.77
25	1.96	1.92	1.89	1.87	1.84	1.82	1.80	1.78	1.76	1.75
26	1.94	1.90	1.87	1.85	1.82	1.80	1.78	1.76	1.74	1.73
27	1.92	1.88	1.86	1.84	1.81	1.79	1.76	1.74	1.72	1.71
28	1.91	1.87	1.84	1.82	1.79	1.77	1.75	1.73	1.70	1.69
29	1.89	1.85	1.83	1.81	1.77	1.75	1.73	1.71	1.69	1.67
30	1.88	1.84	1.81	1.79	1.76	1.74	1.72	1.70	1.67	1.66
35	1.82	1.79	1.76	1.74	1.70	1.68	1.66	1.63	1.61	1.60
40	1.78	1.74	1.72	1.69	1.66	1.64	1.61	1.59	1.56	1.55
50	1.73	1.69	1.66	1.63	1.60	1.58	1.55	1.52	1.50	1.48
60	1.69	1.65	1.62	1.59	1.56	1.53	1.51	1.48	1.45	1.44
70	1.66	1.62	1.59	1.57	1.53	1.50	1.48	1.45	1.42	1.40
80	1.64	1.60	1.57	1.54	1.51	1.48	1.45	1.43	1.39	1.38
90	1.63	1.59	1.55	1.53	1.49	1.46	1.44	1.41	1.38	1.36
100	1.62	1.57	1.54	1.52	1.48	1.45	1.42	1.39	1.36	1.34
120	1.60	1.55	1.52	1.50	1.46	1.43	1.40	1.37	1.33	1.32
150	1.58	1.54	1.50	1.48	1.44	1.41	1.38	1.34	1.31	1.29
200	1.56	1.52	1.48	1.46	1.41	1.39	1.35	1.32	1.28	1.26
250	1.55	1.50	1.47	1.44	1.40	1.37	1.34	1.31	1.27	1.25
300	1.54	1.50	1.46	1.43	1.39	1.36	1.33	1.30	1.26	1.23
400	1.53	1.49	1.45	1.42	1.38	1.35	1.32	1.28	1.24	1.22
500	1.53	1.48	1.45	1.42	1.38	1.35	1.31	1.28	1.23	1.21
600	1.52	1.48	1.44	1.41	1.37	1.34	1.31	1.27	1.23	1.20
750	1.52	1.47	1.44	1.41	1.37	1.34	1.30	1.26	1.22	1.20
1000	1.52	1.47	1.43	1.41	1.36	1.33	1.30	1.26	1.22	1.19