00268

THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics and Philosophy of Engineering

Bachelor of Technology Honors in Engineering

Final Examination (2020/2021)

MHZ5554: Engineering Mathematics IV

Index No: .....

Date: 24^{th} **February 2022** **Time: 0930 hours-1230 hours.**

Instructions:

- This paper consists of TWO(02) Parts: PART A and PART B.
- PART A is Compulsory and consists with TEN(10) short answered questions.
  - Provide short answers in given space.
  - Do not need to show any workings.
- PART B consists with SEVEN(07) questions.
  - Answer any five (05) questions only . All questions carry equal marks.
  - Provide answers in separate sheets (answer booklet) which will be given in the examination.
  - Show all your workings.
  - Answer for each question should commence from a new page.
  - State any assumptions you required.
- Number of pages in the paper is Seven (07).
- All the symbols are in standard notation unless they are defined.
- All the relevant statistical tables are attached with this paper.
- This is a closed book test.
- Attach the Part A to the Answer script of the Part B.
- Do not use red color pen.

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PART A

Provide short answers in the given space. Do not need to show any workings. Please attach this part to your answer script.

1. Determine whether the complex series $\sum_{n=1}^{\infty} \frac{5ni}{(2n-1)}$ is convergent.
   Answer: .....
   ..... *(Marks 1)*

2. The Maclaurin expansion of the complex valued function $f(z) = \frac{1}{(1-3z)}$ is,
   Answer: ..... *(Marks 1)*

3. Find the singular point/s of the complex valued function $f(z) = \frac{e^z - z}{z^2}$ with their order of sigulrity.
   Answer: ..... *(Marks 1)*

4. Let $f : R^2 \to R$ be the function defined as $f(x, y) = \frac{2y}{(x^2-1)}$ for $x \neq \pm 1$.Then ∇f is equal to
   Answer: ..... *(Marks 1)*

5. The circulation of the vector field $\vec{F} = 2xi - 2yj$, along the closed curve $r = 2\cos\theta i + 2\sin\theta j$, where $0 \leq \theta \leq 2\pi$ is,
   Answer: ..... *(Marks 1)*

6. Let the coordinate system $OXYZ$ rotate about the OZ axis by angle $\frac{\pi}{4}$ in the clockwise direction. Then the transformation tensor is,
   Answer: ..... *(Marks 1)*

7. Find the Fourier sine transform of the function $f(x) = \frac{e^{-x}}{x}$, where $x \neq 0$.
   Answer: ..... *(Marks 1)*

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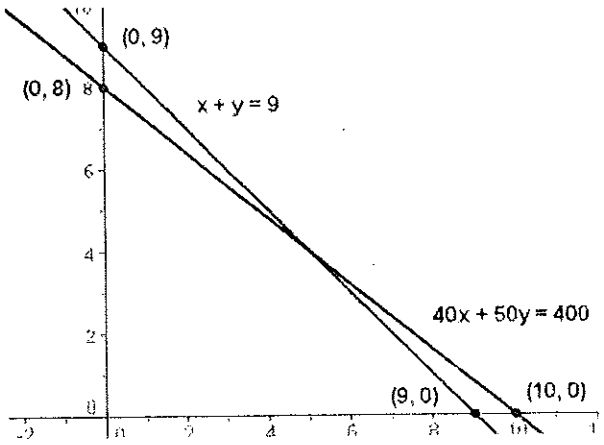
8. Find the points of feasible solutions that graphically represent the constraints in following figure,



Answer: ..... (*Marks 1*)

9. In a Linear programming Problem if the objective function $Z = ax + by$ has the same maximum value on two corner points of the feasible region, then every point on the line segment joining these two points give the same,

Answer: ..... (*Marks 1*)

10. Consider the following summary ANOVA table for a between subjects experiment with 25 total subjects. What can we conclude about the experiment at 5% significant level?

	SS	DF	MSS	F
Between	-	4	-	-
Within	-	20	5	
Total	120	-		

Answer: ..... (*Marks 1*)

PART B

1. (a) Show that $div(gradr^n) = n(n+1)r^{n-2}$, where $r = \sqrt{x^2+y^2+z^2}$. Hence show that $\nabla^2\left(\frac{1}{r}\right) = 0$.

(*Marks 6*)

(b) Find the value of $\iint_S((\vec{F}\cdot\vec{n})dA$, using the Gauss Divergence Theorem over the entire surface of the region above the xy-plane bounded by the cone $z^2 = x^2+y^2$ and the plane $z = 4$, if $\vec{F} = 4xz\boldsymbol{i} + xyz^2\boldsymbol{j} + 3z\boldsymbol{k}$.

(*Marks 12*)

2. (a) By the Green's Theorem area A of the region bounded by a closed curve C is given by,

$$\frac{1}{2}\oint_C (xdy - ydx)$$

Hence find the area of the region in the first quadrant bounded by the curves, $y = x, y = \frac{1}{x}$ and $y = \frac{x}{4}$.

(*Marks 6*)

(b) Verify the Stoke's Theorem for $\vec{F} = (x+y)\boldsymbol{i} + (2x-z)\boldsymbol{j} + (y+z)\boldsymbol{k}$ for the surface of a triangular lamina with vertices $(2,0,0), (0,3,0)$ and $(0,0,6)$.

(*Marks 12*)

3. (a) Find the Laurent series expansion for $F(z) = \frac{2z}{z^2+4}$ around $2i$.

(*Marks 5*)

(b) Use the Residue Theorem to evaluate the following intergation.
$\int_0^{2\pi} \frac{1}{17+8\sin\theta} d\theta$

(*Marks 7*)

(c) Show that the image of the haft plan $A = \{z : Re(z) \geq \frac{1}{4}\}$ under mapping $w = \frac{1}{z}$ is the closed disk $D = \{w : |w-2| \leq 2\}$.

(*Marks 6*)

4. (a) Suppose that there is a rotation of the coordinate system $OXYZ$ through the axis OZ by the angle $\frac{\pi}{3}$ anti-clockwise direction. Consider the vector $\vec{A} = \sqrt{3}\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ which is in rotated coordinate system, where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are the unit vectors with respect to the OX', OY' and OZ' respectively. Then find the vector $\vec{A}$ with respect to the initial coordinate system.

(*Marks 4*)

(b) Determine the mass moment of inertia of a rectangular prism of homogeneous material of density ρ and side lengths a, b, and c about one corner. Select the coordinate system so that its origin is at one corner and let a, b, and c represent the distances along the X, Y, Zaxes, respectively. Construct the symmetric matrix of tensor components referred to this coordinate system, where the components are given by,

$$I_{11} = \int (Y^2 + Z^2)\rho dv, \quad I_{22} = \int (X^2 + Z^2)\rho dv \quad I_{33} = \int (X^2 + Y^2)\rho dv$$

$$I_{12} = -\int (XY)\rho dv, \quad I_{23} = -\int (YZ)\rho dv \quad I_{13} = -\int (XZ)\rho dv$$

If $a = b = c$, then find the symmetric matrix of tensor components derived in the above. Hence find the,

i. Eigen values and eigen vectors of the matrix of tensor components.

(Marks 6)

ii. Matrix of tensor components referred to the principal axes.

(Marks 2)

iii. Verify the result in the above part (b)ii , by using the Transformation Law of tensors.

(Marks 6)

5. (a) Use the method of separation of variables to solve the following partial differential equation,

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

Given that $u = \sin \pi x + \sin 8\pi x - \sin 20\pi x$, when $t \to \infty$, and $u = 0$ at $x = 0$ and $x = 1$.

(Marks 9)

(b) Using the Fourier Transform, solve the partial differential equation given by,

$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2} \quad \text{for } 0 \leq x < \infty, t > 0$$

for the given conditions,

- $u(x, 0) = 0 \quad$ for $x \geq 0$
- $\frac{\partial u}{\partial x}(0, t) = -2$
- $u(x, t)$ is unbounded

(Marks 9)

6. The lifetime of four brands of Flashlight batteries are to be compared by testing each brand in a sample of five flash lights batteries. The lifetimes of the each batteries to the nearest hour are as follows .

Brand A	Brand B	Brand C	Brand D
42	28	24	20
30	36	36	32
39	31	28	38
28	32	28	28
29	27	33	25

Let a group of customers claims that there appear to be a difference in mean lifetime among the four brands of batteries.

(a) Identify the factor variable and dependent variable. *(Marks 2)*

(b) State the null and alternative hypothesises associated with the customers claim. *(Marks 2)*

(c) Test the hypothesises about the customers claim at 5% significant level using ANOVA table and interprete the findings. *(Marks 12)*

(d) State the assumptions you made for this test. *(Marks 2)*

7. A firm produces three products. These products are proceeded on three different machines. The time required to manufacture one units of each of three product and the manufacurering capacity of three machines are given in following table.

Machine	Time per unit(Hours)			Machine Capacity
	Product 1	Product 2	Product 3	(Hours per week)
M1	4	1	1	30
M2	2	3	1	60
M3	1	2	3	40

The profit per unit for product 1, product 2 and product 3 are $Rs.30/=$, $Rs.20/=$ and $Rs.10/=$ respectively. It is assummed that all the products produced are consummed in the market.

(a) Identify the decision variables. *(Marks 2)*

(b) Formulate a mathematical model(Linear Programming Model) to estmate the production structure which will maximize weekly profit.

(Marks 5)

(c) Convert the linear programming problem in part (b) to standard form of the linear programming model. (*Marks 3*)

(d) Wrte down the basic solution of the initial tabular of the simplex method which use to solve the linear programming problem in part (c). (*Marks 2*)

(e) Solve the linear programming problem in part (c) using simplex method. (*Marks 6*)

End.

F Distribution: Critical Values of F (5% significance level)

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.36	246.46	247.32	248.01
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.42	19.43	19.44	19.45
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.71	8.69	8.67	8.66
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.87	5.84	5.82	5.80
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.64	4.60	4.58	4.56
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.96	3.92	3.90	3.87
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.53	3.49	3.47	3.44
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.24	3.20	3.17	3.15
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.03	2.99	2.96	2.94
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.86	2.83	2.80	2.77
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.74	2.70	2.67	2.65
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.64	2.60	2.57	2.54
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.55	2.51	2.48	2.46
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.48	2.44	2.41	2.39
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.42	2.38	2.35	2.33
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.37	2.33	2.30	2.28
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.33	2.29	2.26	2.23
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.29	2.25	2.22	2.19
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.26	2.21	2.18	2.16
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.22	2.18	2.15	2.12
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.20	2.16	2.12	2.10
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.17	2.13	2.10	2.07
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.15	2.11	2.08	2.05
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.13	2.09	2.05	2.03
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.11	2.07	2.04	2.01
26	4.22	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.09	2.05	2.02	1.99
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.08	2.04	2.00	1.97
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.06	2.02	1.99	1.96
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.05	2.01	1.97	1.94
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.04	1.99	1.96	1.93
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.04	1.99	1.94	1.91	1.88
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.95	1.90	1.87	1.84
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.95	1.89	1.85	1.81	1.78
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.86	1.82	1.78	1.75
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97	1.89	1.84	1.79	1.75	1.72
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.88	1.82	1.77	1.73	1.70
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94	1.86	1.80	1.76	1.72	1.69
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.85	1.79	1.75	1.71	1.68
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.78	1.73	1.69	1.66
150	3.90	3.06	2.66	2.43	2.27	2.16	2.07	2.00	1.94	1.89	1.82	1.76	1.71	1.67	1.64
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.80	1.74	1.69	1.66	1.62
250	3.88	3.03	2.64	2.41	2.25	2.13	2.05	1.98	1.92	1.87	1.79	1.73	1.68	1.65	1.61
300	3.87	3.03	2.63	2.40	2.24	2.13	2.04	1.97	1.91	1.86	1.78	1.72	1.68	1.64	1.61
400	3.86	3.02	2.63	2.39	2.24	2.12	2.03	1.96	1.90	1.85	1.78	1.72	1.67	1.63	1.60
500	3.86	3.01	2.62	2.39	2.23	2.12	2.03	1.96	1.90	1.85	1.77	1.71	1.66	1.62	1.59
600	3.86	3.01	2.62	2.39	2.23	2.11	2.02	1.95	1.90	1.85	1.77	1.71	1.66	1.62	1.59
750	3.85	3.01	2.62	2.38	2.23	2.11	2.02	1.95	1.89	1.84	1.77	1.70	1.66	1.62	1.58
1000	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89	1.84	1.76	1.70	1.65	1.61	1.58

F Distribution: Critical Values of F (5% significance level)

ν_2 \ ν_1	25	30	35	40	50	60	75	100	150	200
1	249.26	250.10	250.69	251.14	251.77	252.20	252.62	253.04	253.46	253.68
2	19.46	19.46	19.47	19.47	19.48	19.48	19.48	19.49	19.49	19.49
3	8.63	8.62	8.60	8.59	8.58	8.57	8.56	8.55	8.54	8.54
4	5.77	5.75	5.73	5.72	5.70	5.69	5.68	5.66	5.65	5.65
5	4.52	4.50	4.48	4.46	4.44	4.43	4.42	4.41	4.39	4.39
6	3.83	3.81	3.79	3.77	3.75	3.74	3.73	3.71	3.70	3.69
7	3.40	3.38	3.36	3.34	3.32	3.30	3.29	3.27	3.26	3.25
8	3.11	3.08	3.06	3.04	3.02	3.01	2.99	2.97	2.96	2.95
9	2.89	2.86	2.84	2.83	2.80	2.79	2.77	2.76	2.74	2.73
10	2.73	2.70	2.68	2.66	2.64	2.62	2.60	2.59	2.57	2.56
11	2.60	2.57	2.55	2.53	2.51	2.49	2.47	2.46	2.44	2.43
12	2.50	2.47	2.44	2.43	2.40	2.38	2.37	2.35	2.33	2.32
13	2.41	2.38	2.36	2.34	2.31	2.30	2.28	2.26	2.24	2.23
14	2.34	2.31	2.28	2.27	2.24	2.22	2.21	2.19	2.17	2.16
15	2.28	2.25	2.22	2.20	2.18	2.16	2.14	2.12	2.10	2.10
16	2.23	2.19	2.17	2.15	2.12	2.11	2.09	2.07	2.05	2.04
17	2.18	2.15	2.12	2.10	2.08	2.06	2.04	2.02	2.00	1.99
18	2.14	2.11	2.08	2.06	2.04	2.02	2.00	1.98	1.96	1.95
19	2.11	2.07	2.05	2.03	2.00	1.98	1.96	1.94	1.92	1.91
20	2.07	2.04	2.01	1.99	1.97	1.95	1.93	1.91	1.89	1.88
21	2.05	2.01	1.98	1.96	1.94	1.92	1.90	1.88	1.86	1.84
22	2.02	1.98	1.96	1.94	1.91	1.89	1.87	1.85	1.83	1.82
23	2.00	1.96	1.93	1.91	1.88	1.86	1.84	1.82	1.80	1.79
24	1.97	1.94	1.91	1.89	1.86	1.84	1.82	1.80	1.78	1.77
25	1.96	1.92	1.89	1.87	1.84	1.82	1.80	1.78	1.76	1.75
26	1.94	1.90	1.87	1.85	1.82	1.80	1.78	1.76	1.74	1.73
27	1.92	1.88	1.86	1.84	1.81	1.79	1.76	1.74	1.72	1.71
28	1.91	1.87	1.84	1.82	1.79	1.77	1.75	1.73	1.70	1.69
29	1.89	1.85	1.83	1.81	1.77	1.75	1.73	1.71	1.69	1.67
30	1.88	1.84	1.81	1.79	1.76	1.74	1.72	1.70	1.67	1.66
35	1.82	1.79	1.76	1.74	1.70	1.68	1.66	1.63	1.61	1.60
40	1.78	1.74	1.72	1.69	1.66	1.64	1.61	1.59	1.56	1.55
50	1.73	1.69	1.66	1.63	1.60	1.58	1.55	1.52	1.50	1.48
60	1.69	1.65	1.62	1.59	1.56	1.53	1.51	1.48	1.45	1.44
70	1.66	1.62	1.59	1.57	1.53	1.50	1.48	1.45	1.42	1.40
80	1.64	1.60	1.57	1.54	1.51	1.48	1.45	1.43	1.39	1.38
90	1.63	1.59	1.55	1.53	1.49	1.46	1.44	1.41	1.38	1.36
100	1.62	1.57	1.54	1.52	1.48	1.45	1.42	1.39	1.36	1.34
120	1.60	1.55	1.52	1.50	1.46	1.43	1.40	1.37	1.33	1.32
150	1.58	1.54	1.50	1.48	1.44	1.41	1.38	1.34	1.31	1.29
200	1.56	1.52	1.48	1.46	1.41	1.39	1.35	1.32	1.28	1.26
250	1.55	1.50	1.47	1.44	1.40	1.37	1.34	1.31	1.27	1.25
300	1.54	1.50	1.46	1.43	1.39	1.36	1.33	1.30	1.26	1.23
400	1.53	1.49	1.45	1.42	1.38	1.35	1.32	1.28	1.24	1.22
500	1.53	1.48	1.45	1.42	1.38	1.35	1.31	1.28	1.23	1.21
600	1.52	1.48	1.44	1.41	1.37	1.34	1.31	1.27	1.23	1.20
750	1.52	1.47	1.44	1.41	1.37	1.34	1.30	1.26	1.22	1.20
1000	1.52	1.47	1.43	1.41	1.36	1.33	1.30	1.26	1.22	1.19