THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Bachelor of Software Engineering Honors

Final Examination (2019/2020)
MHZ4256: Mathematics for Computing

Date: 28th July 2020 (Tuesday) **Time: 13:30 – 16:30**

Instruction:

- **Answer only six questions.**
- **Please answer a total of six questions choosing at least one from each single section.**
- **All symbols are in standard notation and state any assumption that you made.**

SECTION – A

Q1.

I. Decide which of the following are propositions. What are the truth values of those that are propositions? **[20%]**

a) "$x^2 \geq 10$" ;
b) "$5 + 7 = 1\ 0$";
c) "Colombo is in England or $1 + 9 = 8$".
d) "London is the capital of India".

II. State the "convers", "inverse", and "contrapositive" of each of the following statement: **[30%]**

a) If x is an integer and x^2 is odd, then x is odd;
b) If 11 pigeons live in 10 birdhouses, then there are two pigeons that live in the same birdhouse.

III. Let p, q, and r be three statements.
Verify that each of the following statement is a tautology or not. **[30%]**

a) $((p \to q) \to r) \to (p \to (q \to r))$
b) $(p \to (r \vee q)) \to ((p \to r) \vee (p \to q))$

IV. Determine the truth value and Negation of the each of the following statements: **[20%]**

a) $\forall x \in \mathbb{R},\ (x^2 \geq x)$;
b) $\exists x \in \mathbb{R}, (x = 1)$.

Q2.

I. Test the validity of the following argument:

If I want to be a lawyer, then I want to study logic.
If I don't want to be a lawyer, then I don't like to argue.

Therefore, If I like to argue, then I want to study logic. **[25%]**

II. By using truth tables, prove De.Morgan laws of propositions. **[30%]**

III. Using Mathematical induction, for a positive integer n, prove of the following:

$\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$ for all $n \geq 1$. **[25%]**

IV. By giving a counter example, disprove each of the following statements: **[20%]**

a) all odd number are divisible by 3;
b) the square root of any integer is irrational.

Q3.

I. Consider the following truth table. **[30%]**

A	B	F
0	0	1
0	1	1
1	0	0
1	1	1

Where A and B are input variables and F is an output.

a) Create a Karnaugh map (K-map) based on the above truth table.
b) Using the K-map created in the above part I.(a), write down the Boolean function.

II. Consider the following Boolean function given below. **[25%]**

$$F = A + \bar{A}B$$

Create a K-map based on the above Boolean function and draw the truth table.

III. Minimize the following Boolean function by Algebraic method. **[45%]**

a) $F_1 = AB + \bar{A}CD + \bar{B}CD$

b) $F_2 = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + ABC + A\bar{B}C$

c) $F_3 = \bar{A} + A\bar{B} + \overline{(\bar{A} + \bar{B})}C$

SECTION – B

Q4.

I. Write down the elements in each of the following set: **[20%]**

a) $A = \{x: x^4 - 5x^2 + 6 = 0,\ x \in \mathbb{R}^+\}$;

b) $B = \{x: x \geq 14,\ x = 2n + 1,\ n \in \mathbb{Z}^+\}$;

c) $C = \{x: x^2 + 1 = 10,\ x \in \mathbb{N}\}$,

d) $D = \{x: |x + 3| \leq 6,\ x \in \mathbb{Z}^+\}$

II. $L = \{9, 19, 29, 39, 49\}$, $M = \{39, 49, 59, 69\}$, $N = \{59, 69, 79, 89\}$. Find

a) $L \oplus M$;

b) $M \oplus N$;

c) $L \cap (M \oplus N)$, where $\oplus$ is symmetric difference. **[15%]**

III.

a) Define the Cartesian product of set A and B. **[05%]**

b) $A = \{a, b, ab, ba\}$ and $B = \{1,\ 12, 21\}$. Find $A \times B$ and A^2. **[20%]**

IV. Let $E = \{e, d,\ \text{ed},\ de\}$. Find the power set $P(E)$ of E. **[10%]**

V. Without using Venn diagram, Show that

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. **[30%]**

Q5.

I. Let $h: \mathbb{R} \to \mathbb{R}$ be defined by

$$h(x) = \begin{cases} x + 7\ ; & x \leq 0 \\ -2x + 5; & 0 < x < 3\ . \\ x - 1; & x \geq 3 \end{cases}$$

Find $h(-3)$, $h(0)$, and $h(3)$. **[30%]**

II. Let $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = ax^2 + b$ and $g(x) = cx^2 + d$ respectively for all $x \in \mathbb{R}$, where $a, b, c,$ and d are constants with $a \neq 0$ and $c \neq 0$.
Find the relationship(s) between the constant $a, b, c, d,$ $if\ f \circ g(x) = g \circ f(x)$ for all x. **[30%]**

III. Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Define $m(x) = \frac{x-2}{x-3}$. Prove that $m(x)$ is invertible and find a formula for $m^{-1}(x)$. **[40%]**

Q6.

I. Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5\}$. In each of the following, find all the pairs of A x B that belong to R_1 and R_2.

a) $R_1 = \{(x, y) | x \geq y;\ x \in A, y \in B\}$ **[10%]**

b) $R_2 = \{(x, y) | x^2 = y; x \in A, y \in B\}$. **[10%]**

II.

a) Define the equivalence relation by the usual notation. **[10%]**

b) Determine whether the following relations are equivalence relation or not.

α) Let A be a set if integers and R_3 be the relation of $A \times A$ defined by $(a, b)R_3(c, d)$ if $ad = bc$. **[25%]**

β) If R_2 be the relation which is defined by " aR_2b iff $a - b$ is divisible by 10" for $a, b \in \mathbb{Z}$. **[25%]**

III. Show that "$x \leq y$" is a partial order relation in $\mathbb{R}$, where $x, y \in \mathbb{R}$. **[20%]**

SECTION – C

Q7.

I. Separate following into real and imaginary parts: **[30%]**

a) $(3 + i4) + i(4 + i5) + (2 + i3)^2$,

b) $\frac{3+i4}{5+i7}$,

c) $\frac{2-i5}{(2-i3)^2}$.

II. Solve the following equation: **[20%]**

$x^2 + 2x + 5$.

III. If $z = x + iy$ and $z \neq 0$, then define the polar form of a complex number z. **[20%]**

IV. Find the modulus, principle argument and argument of the following complex numbers. **[30%]**

a) $1 + i$

b) – 4i

c) 4+ 3i

Q8.

I. Let $z \in \mathbb{C}$. Prove that following: **[20%]**

a) $2Re(z) = z + \bar{z}$,

b) $Im\, z = \frac{z-\bar{z}}{2i}$

II. Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$.

Show that, **[30%]**

a) $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$,

b) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$,

Where $x_1, x_2, y_1, y_2 \in \mathbb{R}$.

III. Let $z \in \mathbb{C}$. Show that $z\bar{z} = |z|^2$. **[10%]**

IV. Find the modulus of (3 + 4i) (5 + 12i). **[10%]**

V. If 3 + 2i, 5+ 7i, 5 + 8i represent the points P, Q, R respectively on an Argand diagram. Find the complex number which represents the point S such that PQRS is a parallelogram and the lengths of the diagonals. **[30%]**

Q9.

I. ABCD is a square with center at the origin. If A represents the complex number 5 - 4i find the complex numbers which represent the other vertices. Also find the lengths of a diagonal and a side. **[25%]**

II. Draw the locus of the following: **[25%]**

a) $\arg(z + 2) = {}^{\pi}/_{6}$,

b) $\arg(3 - 5z) = {}^{\pi}/_{3}$.

III. Sketch the following sets in the complex plane. **[50%]**

a) $\left\{z \in \mathbb{C}: \frac{\pi}{4} \leq arg(z) \leq \frac{\pi}{3} = 4\right\}$.

b) $\{z \in \mathbb{C}: |z + 2| + |z - 2| = 5\}$.

c) $\left\{z \in \mathbb{C}: Im\left(\frac{2z+1+3i}{2z-1}\right) = 0\right\}$.

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