The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Electrical & Computer Engineering

Study Programme	: Bachelor of Software Engineering Honors
Name of the Examination	: Final Examination
Course Code and Title	**: EEZ3562/ECZ3262 – Mathematics**
Academic Year	: 2019/2020
Date	: 26th July 2020
Time	: 1330-1630hrs
Duration	: **3 hours**

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Eight (8)** questions in **Six (6)** pages.
3. Answer any **Five** out of eight questions. All question carry equal marks.
4. Show all steps clearly.
5. Answer for each question should commence from a new page.
6. This is a Closed Book Test **(CBT).**
7. **Programmable** calculators are not allowed.
8. Do not use red color pen.

Q1

a) Using Demorgan's theorem, simplify the following expressions. [6]

i. $\overline{(\bar{a}+b)(a+\bar{b})}$

ii. $\overline{(a+b+c)abc}$

b) Let p, q and r be propositions. By constructing the truth tables, show the following propositional equivalencies. [4]

i. $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

ii. $(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

c) Consider the following truth table. [10]

A	B	C	D	Result
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

i. Setup the Karnaugh map for the above truth table.

ii. Then find the solution and simplify using the K map.

Q2

a) If $A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$, then show that [6]

$A^2 = 3A - 2I$; where I is the identity matrix of order 2.

b)

i. Let $A = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}$. Is the matrix A nilpotent? Justify your answer. [3]

ii. If $A = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 7 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 6 \\ 1 & 4 \\ 5 & 2 \end{bmatrix}$ then find AB. [3]

c) Using the method of Gaussian elimination, solve the following system of linear equations.

[8]

$$4x + 8y - 4z = 4$$
$$3x + 8y + 5z = -11$$
$$-2x + y + 12z = -17$$

Q3

a) Using the first principles, find the first derivatives of the following. [6]

i. $y = x^2 + 2$

ii. $y = Sinx + 1$

b) Find $\frac{dy}{dx}$ of the following functions. [8]

i. $y = \frac{1}{3}(\sqrt{1 + x^4} - x^2)^3$

ii. $y = x^2 sinx$

c) If $y = -3x - \frac{1}{2}sin2x + 4cosx$, then show that [6]

$\frac{dy}{dx} = -6 + 2(sinx - 1)^2$

Q4

a) Find the following indefinite integral. [6]

$$\int \frac{1}{x^3 - 1} dx$$

b) Using the integration by parts, find the following indefinite integral. [8]

$$\int sinxln(cosx)dx$$

c) Evaluate the following definite integral. [6]

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} sin^3 xcosxdx$$

Q5

a) For the statistical distribution of given below, calculate the **Mode, Median, Mean, Range, Variance** and **Standard Deviation.**

[6]

xi	61	64	67	70	73
f_i	5	18	42	27	8

b) The following table shows the recorded high temperatures for each of the 50 engines. Construct a histogram and a frequency polygon to represent the data. [8]

Class Boundaries	**Frequency**
99.5 – 104.5	2
104.5 – 109.5	8
109.5 – 114.5	18
114.5 – 119.5	13
119.5 – 124.5	7
124.5 – 129.5	1
129.5 – 134.5	1

c) XYZ (Pvt) Ltd has a contract to assemble components for a waste water management system to be used by the government. The time required to complete one part of the assembly is thought to be normally distributed, with a mean of 30 hours and a standard deviation of 4.7 hours. Find the probability that the assembly steps completed between 26 and 35 hours. [6]

Q6

a) Evaluate the following limits.

(i) $\lim_{x\to 0} \frac{\sin 5x}{-3x}$. (ii) $\lim_{x\to 9} \frac{2x^2-162}{\sqrt{x}-3}$. (iii) $\lim_{x\to -2} \frac{x^3+8}{\frac{1}{2}+\frac{1}{x}}$.

[6]

b) Let X and $\overline{X}$ are the exact value and the computed value of an answer, respectively. Find the **absolute error** and the **relative error** when: [4]

(i) $X = -0.0047, \overline{X} = -0.0045$

(ii) $X = -0.671 \times 10^{12}, X = -0.0669 \times 10^{13}$

c) The difference table for $f(x) = e^x$ with h = 0.2 is shown below. [10]

x_i	f_i	Δf_i	$\Delta^2 f_i$	$\Delta^3 f_i$	$\Delta^4 f_i$
0.0	1.0000				
		0.2214			
0.2	1.2214		0.0490		
		0.2704		0.0109	
0.4	1.4918		0.0599		0.0023
		0.3303		0.0132	
0.6	1.8221		0.0731		0.0031
		0.4034		0.0163	
0.8	2.2255		0.0894		0.0033
		0.4928		0.0196	
1.0	2.7183		0.1090		0.0047
		0.6018		0.0243	
1.2	3.3201		0.1333		
		0.7351			
1.4	1.0552				

Selecting $x_0 = 0.4$ and using **Newton's forward** formula, find the value of $f(0.43)$.

Q7

a) Given that $\tan\alpha = \frac{1}{2}$ and $\pi < \alpha < \frac{3\pi}{2}$, and $\sin\beta = \frac{3}{4}$ and $\frac{\pi}{2} < \beta < \pi$. Find,

Sin 2α ii) cos(α+β) iii) tan(α - β) [6]

b) Sketch the graph of $y = sin^2x$, where $-2\pi \leq x \leq 2\pi$. [6]

c) Find the height of a chimney when it is found that, on walking towards it 50m on a horizontal line through its base, the angular elevation of its top changes from 30° to 45°. [8]

Q8

a) A pyramid has a square base with side length of $8\,m$. The four lateral faces are congruent isosceles triangles with lateral edges of length $10\,m$. Find the surface area of this pyramid. [10]

b) A trapezoid $ABCD$ with AD parallel to BC has angle D equal to 40^0, the length of DC is equal to $2\,m$, the length of BC is equal to $5\,m$ and the area of the trapezoid is equal to $20\,m^2$. Calculate the length of AD. [10]