

The Open University of Sri Lanka  
Faculty of Engineering Technology  
Department of Electrical & Computer Engineering



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| Study Programme         | : Bachelor of Software Engineering Honors    |
| Name of the Examination | : Final Examination                          |
| Course Code and Title   | : <b>MHZ3459 – Mathematics for Computing</b> |
| Academic Year           | : 2019/2020                                  |
| Date                    | : 26 <sup>th</sup> July 2020                 |
| Time                    | : 1330-1630hrs                               |
| Duration                | : <b>3 hours</b>                             |

1. Read all instructions carefully before answering the questions.
  2. This question paper consists of **Eight (8)** questions in **Five (5)** pages.
  3. Answer any **five** out of eight questions.
  4. Show all steps clearly
  5. Answer for each question should commence from a new page.
  6. This is a Closed Book Test (**CBT**).
  7. **Programmable** calculators are not allowed.
  8. Do not use red color pen.
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Q1

a) Let  $A, B, C$  and  $D$  be Boolean variables. Prove the following identities [6]

i.  $AB + \overline{A}C + BC = AB + \overline{A}C$

ii.  $(\overline{AB})(\overline{A+B})(\overline{B+B}) = \overline{A}$

iii.  $\overline{AB + B(C + \overline{CD})} = \overline{AB} + \overline{AB} + \overline{ACD} + BCD$

b) Using Truth Tables show the following: [4]

i.  $\overline{(\overline{a} + \overline{b})(\overline{a} + b)} = a$

ii.  $x + \overline{y} + z = \overline{x}y\overline{z}$

c) Consider the following truth table. [10]

| A | B | C | D | Result |
|---|---|---|---|--------|
| 0 | 0 | 0 | 0 | 1      |
| 0 | 0 | 0 | 1 | 1      |
| 0 | 0 | 1 | 0 | 1      |
| 0 | 0 | 1 | 1 | 1      |
| 0 | 1 | 0 | 0 | 0      |
| 0 | 1 | 0 | 1 | 0      |
| 0 | 1 | 1 | 0 | 0      |
| 0 | 1 | 1 | 1 | 0      |
| 1 | 0 | 0 | 0 | 0      |
| 1 | 0 | 0 | 1 | 0      |
| 1 | 0 | 1 | 0 | 1      |
| 1 | 0 | 1 | 1 | 1      |
| 1 | 1 | 0 | 0 | 0      |
| 1 | 1 | 0 | 1 | 0      |
| 1 | 1 | 1 | 0 | 1      |
| 1 | 1 | 1 | 1 | 1      |

- i. Setup the Karnaugh map for the above truth table.
- ii. Then find the solution and simplify using the Karnaugh map.

Q2

a) If  $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$ , then show that [6]

$A^2 + 2I = A$ ; where  $I$  is the identity matrix of order 2.

b)

Let  $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{bmatrix}$ , show that  $A^2 = A$  [8]

Hence, deduce that  $(I - A)^2 = (I - A)$ , where  $I$  is the identity matrix of order 3.

c)

Let  $A = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

Show that  $A^2 = I$ , where  $I$  is the identity matrix of order 3. [6]

Q3

If  $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ , then

a) Find  $AA^T$  [6]

b) Using Gaussian elimination method, find the inverse of the matrix  $A$ . [14]

Q4

a) Given that  $\sin \theta = \frac{3}{8}$  and  $0 < \theta < \frac{\pi}{2}$ , and  $\sin \alpha = \frac{5}{13}$  and  $\frac{\pi}{2} < \alpha < \pi$ . Find

i.  $\sin(\theta - \alpha)$       ii.  $\cos(\theta + \alpha)$       iii.  $\tan(\theta - \alpha)$

[12]

b) Sketch the graph of  $y = \cos^2 x$ , where  $-2\pi \leq x \leq 2\pi$  [8]

Q5

a) Write the formula for  $\sin(A + B)$  [6]

i. Using the above formula, prove that  $\sin 3A = 3\sin A - 4\sin^3 A$ .

ii. Hence, find the value of  $\sin 135^\circ$

b) Prove the following trigonometric identities

[9]

i.  $(1 - \cos^2 x)(1 + \cos^2 x) = 2\sin^2 x - \sin^4 x$

ii.  $\frac{2\cos^2 x}{2\cot x - \sin 2x} = \tan x$

iii.  $\sin 75^\circ + \sin 15^\circ = \sqrt{\frac{3}{2}}$

c) Find the general solution of the following equation

$$2\sin^2 x - 3\sin x + 1 = 0 \text{ in the range.}$$

[5]

Q6

a) Evaluate the following limits.

i.  $\lim_{x \rightarrow 0} \frac{\tan x - x}{\sin x}$

ii.  $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x^2 - 2x - 3}$

iii.  $\lim_{x \rightarrow 1} \frac{1 - \cos x}{x^2}$

[6]

b) Differentiate the following functions with respect to  $x$ .

i.  $y = e^{-2x} \sin 2x$

ii.  $y = \left( \frac{x \sin x}{1 + \cos x} \right)$

[8]

c) Find the equation of the tangent to the curve  $y = \sin(3x) + 1$  at the point where  $x =$

$$\frac{\pi}{3}$$

[6]

Q7

a) Using the first principles, find the first derivative of the following.

[12]

i.  $y = x^2 + 3x + 2$

ii.  $y = \frac{1}{1+x}$

iii.  $y = \sin x + 1$

iv.  $y = \cos x$

b) Taking  $x_0 = 1$  as an initial approximation for the root of Newton-Raphson formula, obtain two further approximations to the positive root of  $x^2 - 3 = 0$ , giving your answers to 3 decimal places.

[8]

Q8

a) Find the following indefinite integrals.

[6]

i.  $\int (\sin 2x - \cos 3x) dx$

ii.  $\int \sin(1 - x) dx$

b) Evaluate the following definite integrals.

[8]

i.  $\int_0^2 \frac{1}{x^2+4} dx$

ii.  $\int_0^3 \frac{1}{\sqrt{9-x^2}} dx$

[6]

c)

i. Sketch the curves of the graph of  $y = x^2$  and the straight line  $y = x$  in the same figure.

ii. Find the coordinates of the points of intersection of the curves.

iii. Hence, find the area bounded by the two graphs  $y = x^2$  and  $y = x$ .

