THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Bachelor of Technology Honors in Engineering / Bachelor of Software Engineering Honors

Final Examination (2019/2020)
MHZ5340 / MHZ5360 / MPZ5140 / MPZ5160: Discrete Mathematics II

Date: 02nd August 2020 (Sunday) **Time: 13:30 – 16:30**

Instruction:

- **Answer only six questions.**
- **Please answer a total of six questions choosing at least one from each single section.**
- **All symbols are in standard notation and state any assumption that you made.**

SECTION – A

Q1.

i. Let A be any nonempty set with the operation " $*$ " defined by $x * y = 2(x + y) + 3xy$. Is the operation,
   a. Associative?
   b. Commutative?

   Justify your answer. [30%]

ii. Define an abelian group in usual notation. Prove that $\mathbb{R}\backslash\{-\frac{1}{2}\}$ is an abelian group with respect to the binary operation defined on $\mathbb{R}\backslash\{-\frac{1}{2}\}$ by $x * y = x + y + 2xy$. [45%]

iii. Define a semi group in usual notation. Let " $*$ " be operation on $\mathbb{R}$ defined by the following way:

$$x * y = 5(x + y)$$

Find that whether $(\mathbb{R}, *)$ is a semi group. [25%]

Q2.

i. Let $G = \left\{\begin{bmatrix} a & b \\ 0 & d \end{bmatrix} \middle| \ a, b, d \in \mathbb{Z}, ad \neq 0\right\}$. Show that G is a group under usual matrix multiplication. [60%]

ii. Let $(G,*)$ be a group. If $a, b \in G$. Suppose that $a * b = b * a^{-1}$ and $b * a = a * b^{-1}$. Show that $a^4 = b^4 = e$. [40%]

Q3.

i. Define a homomorphism and Isomorphism for group in usual notation. **[20%]**

ii. For a fixed element in a group G, define $f_a: G \to G$ by
$f_a(x) = a^{-1}\, x\, a,$ for all $x \in G$. Show that f_a is a homomorphism. **[40%]**

iii. Let $G = \left\{ \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \middle| \; a \in \mathbb{R} \right\}$, and $G' = \mathbb{R}$. Assuming that G and G' are groups under the usual matrix multiplication. Let the mapping $\phi: G \longrightarrow G'$, defined by $\phi(A) = a$ where $A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$ for all $A \in G$.
Show that ϕ is an Isomorphism. **[40%]**

SECTION – B

Q4.

i. Define a simple graph. **[15%]**

ii. By drawing each of the following graph, determine whether which of those graphs are simple or not. **[45%]**

a. $G_1 = \{V_1, E_1\}$ where $V_1 = \{1, 2, 3, 4, 5, 6\}$ and
$E_1 = \{\{x, y\}\; 2x + y\ is\ even\ and\ x \leq y\}$

b. $G_2 = \{V_2, E_2\}$ where $V_2 = \{1, 2, 3, 4, 5, 6{,}7\}$ and
$E_2 = \{\{1{,}2\}, \{1{,}3\}, \{1{,}4\}, \{2{,}5\}, \{2{,}6\}, \{3, 6\}, \{4, 5\}, \{4{,}7\}\}$

c. $G_3 = \{V_3, E_3\}$ where $V_3 = \{1, 2, 3, 4, 5, 6\}$ and
$E_3 = \{\{i, j\}\; i \times j\ is\ a\ divided\ by\ 2\ and\ i < j\}$

iii. Let G be a graph of 8 vertices and 13 edges in which every vertex is of degree 3 or 4. How many vertices of degree 3 and 4 does G have? Construct one such graph G. **[40%]**

Q5.

i. Define a tree graph and the forest. **[10%]**

ii. Is it possible to draw the each of the following case: **[40%]**

a. A tree with 11 vertices, each of which has either degree 1 or degree 3.
b. A tree with 13 vertices, exactly 9 of which have degree of one.

iii. Suppose that Ajith, Bandara, Chamara, Dias, Erika, Fernando and Geetha are planning a quiz. They are assigned to work on the following subjects: **[50%]**

A – Calculus: Ajith, Chamara, Dias

B – Analysis: Chamara, Fernando

C – Algebra: Fernando, Ajith

D – History: Bandara, Dias, Erika

E – English: Erika, Geetha

F – Psychology: Erika, Fernando, Geetha

They want to know how many meeting times are necessary in order for each subject to meet once. Let the vertices of a graph be A, B, C, D, E, F and the edges of a graph represent common members in subjects.

a. Draw the graph representing the above relationship.
b. Find the degree for each vertex of the graph.
c. Write down the adjacency matrix for the graph.

Q6.

i. Define the connected graph. **[10%]**

ii. G is the graph whose adjacency matrix A is given by **[50%]**

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

a) Without drawing a graph of G, determine whether G is connected or not.
b) If $V(G) = \{a, b, c, d\}$ then find the number of paths of length four joining vertices d and b.
c) Draw the graph of a adjacency matrix A.

iii. Let $G = G(V, E)$ be a connected graph with at least two vertices and suppose that $|E(G)| < |V(G)|$. Prove that, G has at least one vertex of degree one. **[40%]**

SECTION – C

Q7.

i. Iterate the Eco-system growth for the relation $k_{n+1} - \lambda k_n = 0$ for $k_0 = 0.4$, taking $\lambda = 0.4$ and $\lambda = 1.4$, and draw the diagram. Hence deduce k_n as $n \to \infty$. (At least 5 iteration steps are necessary) **[40%]**

ii. Iterate the Eco-system growth model relationship $y_{n+1} - \lambda y_n + \lambda y_n^2 = 0$, where $\lambda = 2.5$ and $y_0 = 0.2$ and draw the diagram. Hence deduce k_n as $n \to \infty$. (At least 5 iteration steps are necessary) **[30%]**

iii. Draw the graph for the relation $Z_{n+1} = Z_n^2$, where $Z_0 = 1.5 + 0.2i$. Find Z_5 and hence deduce Z_n as $n \to \infty$. **[30%]**

Q8.

A three-dimensional system is governed by the following system of differential equations:

$$\frac{dx}{dt} = 3x + 2y - 2z$$

$$\frac{dy}{dt} = 2x + 3y - 2z$$

$$\frac{dz}{dt} = -2x - 2y + 3z$$

where x, y and z are function of t and at $t = 0, (x, y, z) = (1, 0, 1)$.
Find the phase space value $x(t), y(t), z(t)$ for $t = 1, 2$. **[100%]**

Q9.

I. Let $L_1 = \{1, 11, 111\}$ and $L_2 = \{2, 22,222\}$ be languages. Find
   a) $L_1 L_2$, **[10%]**
   b) $L_2 L_1$. **[10%]**

II. Show that the string $((-t * t) + (tnt))$ is a sentence generated by the grammar G, where, $G = \{\{S, K\}, \{+, *, -, (\,), t, n\}, P, S\}$ and starting symbol S and production P. **[25%]**

$$P = \{S \to SKS, S \to t, S \to (S), S \to S * S, S \to -S, K \to +, K \to -, K \to n\}.$$

III. Draw the directed graph that describers the DFA (Deterministic Finite Automation) with the following state transition table. **[25%]**

State	Input				Output			
	a	b	c	d	a	b	c	d
s_0	s_0	s_1	s_2	s_1	1	0	0	0
s_1	s_1	s_0	s_1	s_2	1	1	1	1
s_2	s_0	s_3	s_2	s_1	0	1	0	0
s_3	s_2	s_3	s_1	s_4	1	0	1	0
s_4	s_4	s_4	s_4	s_4	0	1	1	1

Initial state s_0 and accepting state s_4.

IV. Draw Let M be a Mealy machine. Let $s \in S, a, b \in I$ and $s \in I^*$ and defined functions. **[30%]**

$\delta: S \times I^* \to S$ and $\beta^*: S \times I^* \to O^*$ by

$\delta^*(s, \Omega) = s,$

$\delta^*(s, a.x^/) = \delta^*[\delta(s,a), x^/],$

$\beta^*(s, \Omega) = \Omega,$

$\beta^*(s, a.x^/) = \beta(s,a)\beta^*[\delta(s,a), x^/].$

The two-frame binary pipeline devise hold up two binary as in the following table,

State	Input			Output		
	a	**b**	**c**	**a**	**b**	**c**
00	11	10	10	0	1	1
01	01	00	01	1	0	1
10	01	10	11	0	1	0
11	11	10	00	1	0	1

Find the two-frame binary pipeline buffer and work out its response to the sequence *aacbcbba* from the state 10.

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