# THE OPEN UNIVERSITY OF SRI LANKA
## Faculty of Engineering Technology
## Department of Mathematics & Philosophy of Engineering

## Bachelor of Industrial Studies

**Final Examination (2020/2021)**
**MHZ3458 Mathematics for Agriculture**

**Date: 1st February 2022 (Tuesday)** **Time: 1400 hrs. – 1700 hrs.**

***Instructions:***

- *Answer* ***five (05)*** *questions only.*
- *Number of pages in the paper is four (04).*
- *All the symbols are in standard notation unless they are defined.*

Q1.

a. Find the factors of the following expressions.

i. $27x^3 + 1.$ (15%)

ii. $x^4 - 16.$ (15%)

b. After expanding, simplify the following expressions.

i. $(x^2 + y^2 + xy)(x^2 + y^2 - xy).$ (15%)

ii. $(a + b)(a + b + c)(a - b)$ (15%)

c. if $a \neq 0$, then solve the following quadratic equation

$$4ax^2 + 5abx + ab^2 = 0$$

Using

i. The formula. (10%)

ii. The completing square method. (10%)

d. Solve the following inequality and represent the solution on the number line.

$$\frac{3}{x-3} < 2$$

(20%)

Q2.

a. Let $f$ be function such that $f(x) = ax^2 + bx + c$ , where $a, b, c \in \mathbb{R}$ such that $a \neq 0, b \neq 0$ and $a - b + c = 0$. Show that $-1$ is a root of the equation $f(x) = 0$. Furthermore, show that 1 is not a root of the above equation. (30%)

b. Let $p(x) = x^3 + 2x^2 + 3x - 1$ and $q(x) = x^2 + 3x + 6$.

   i. Using the remainder theorem, find the remainders when $p(x)$ is divided by $(x - 1)$ and $q(x)$ is divided by $(x - 2)$ respectively. (20%)

   ii. By using the method of long division, verify that $p(x) = (x - 1)q(x) + 5$. (20%)

   iii. Find the remainder when $p(x)$ is divided by $(x - 1)(x - 2)$. (30%)

Q3.

a. Write down the binomial expansion of $(3 + 2x)^6$ in increasing powers of $x$. Let $A_r$ be the term containing $x^r$ in the above expansion for $r = 0, 1, 2, 3, 4, 5, 6$. Show that,

$$\frac{A_{r+1}}{A_r} = \frac{2(6 - r)}{3(r + 1)}x \quad \text{for } x \neq 0.$$

(35%)

b. Find the summations of the first $n$ terms of the following series.

   i. $1 + 4 + 7 + 10 + 13 + \cdots \dots \dots \dots$ (10%)

   ii. $1 + 3 + 3^2 + 3^3 + \cdots \dots \dots \dots$ (10%)

c. Let $\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8\}, A = \{1, 2, 3, 5, 8\}, B = \{1, 2, 3, 4, 6\}$ and $C = \{1, 2, 4, 7\}$. Verify the following results.

   i. $A \cap (B \cap C) = (A \cap B) \cap C$ (15%)

   ii. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (15%)

   iii. $(A \cap B)' = A' \cup B'$ (15%)

Q4.

a. Evaluate the following limit.

$$\lim_{x \to 3} \frac{\sqrt{x - 2} - 1}{\sin(\pi(x - 3))}$$

(25%)

b. Using the first principles, find the first derivative of $y = \sin x$. (25%)

c. Differentiate the following functions with respect to $x$.

i. $y = e^{-2x}\sin 3x$ (25%)

ii. $y = \log\left(\sqrt{\frac{2}{1-x^2}}\right)$ (25%)

Q5.

a. Using the partial fractions, find the following indefinite integral.

$$\int \frac{1}{(x-2)(x+1)}dx$$

(20%)

b. Using a suitable substitution or any other method, find the following indefinite integrals.

i. $\int xe^{-x^2}\,dx$ (20%)

ii. $\int \operatorname{cosec}\theta\, d\theta$ (20%)

c. Using the integration by parts, find the integral $\int xe^x dx$, and hence find the area of the region enclosed by the curve $y = xe^x$ and the lines $x = 1, x = 2$ and $y = 0$. (40%)

Q6.

a. Show that $\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2 = 1 + \sin x$ for $-\pi < x \leq \pi$. Hence show that the value of $\cos\frac{\pi}{8} + \sin\frac{\pi}{8} = \sqrt{\frac{2+\sqrt{2}}{2}}$. (40%)

b. Show that

$$\tan^2\frac{B}{2} = \frac{1-\cos B}{1+\cos B} \quad \text{for } 0 < B < \pi$$

In the usual notation, using the Cosine rule for the triangle $ABC$, show that

$$(a+b+c)(a+c-b)\tan^2\frac{B}{2} = (b+c-a)(a+b-c)$$

(60%)

Q7.

a. Let $\lambda, \mu \in \mathbb{R}$, $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 5 \\ 5 & 3 \end{bmatrix}$ and $X = \begin{bmatrix} \mu \\ \mu + 1 \end{bmatrix}$. Find the values of $\lambda$ and $\mu$ such that $BX = \lambda AX$. Hence deduce that the inverse of $B - \lambda A$ **does not exist.** (40%)

b. If $C = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$, then

i. Find the characteristic equation of $C$. (20%)

ii. Hence, deduce the inverse matrix of $C$. (20%)

c. Using the properties of determinants, prove that

$$\begin{vmatrix} a & a(1-a) & a^2 \\ b & b(1-b) & b^2 \\ c & c(1-c) & c^2 \end{vmatrix} = 0$$

(20%)

Q8.

a. In a parallelogram $ABCD$, $|\overrightarrow{AB}| = 2m$, $|\overrightarrow{AD}| = 1m$ and $B\hat{A}D = \frac{\pi}{3}$. Let $M$ be the mid-point of $CD$. The forces of magnitudes $5, 5, 2, 4$ and $3$ in newtons act along $AB, BC, DC, DA$ and $BM$ respectively.

i. Find the magnitude and the direction of the resultant force. (50%)

ii. Find the distance of the point, where the line of action of the resultant meets $AB$ from the point $A$. (50%)

Q9.

a. A particle is projected from a point $O$ on a horizontal plane with initial velocity $u = \sqrt{2ga}$ at an angle $\theta$ to the horizontal. The particle is just clears a vertical wall of height $\frac{a}{4}$ located at a horizontal distance $a$ from $O$. Show that $\sec^2\theta - 4\tan\theta + 1 = 0$.

(50%)

b. A car of mass $1000kg$ travelling along a horizontal straight road accelerates uniformly from $15ms^{-1}$ to $25ms^{-1}$ in a distance of $320m$. If the resistance to the motion is $145N$, then find the driving force of the engine.

(50%)

*End*