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THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Bachelor of Technology Honors in Engineering

Final Examination (2020/2021)
MHZ3551: Engineering Mathematics I

Date: 19th January 2022 (Wednesday) **Time: 1400 hrs. – 1700 hrs.**

Instructions:

- *Answer five (05) questions only.*
- *Number of pages in the paper is three (03).*
- *All the symbols are in standard notation unless they are defined.*

Q1. a). Let p and q be any two propositions. By using laws in propositional logic, show that the proposition, $(p \Rightarrow q) \Rightarrow \neg p$ is equivalent to $\neg(p \wedge q)$. (25%)

b) Let φ be the formula defined as $\forall x \forall y \exists z(x > y \Rightarrow x > z > y)$ for $x, y, z \in \mathbb{R}$. Then prove that $\neg\varphi$ is equivalent to $\exists x \exists y \forall z(x > y \wedge (z \geq x \vee y \geq z))$. (25%)

c) Let $a \in \mathbb{Z}$. Use the proof by case to show that if a is not divisible by 3, then $a^2 + 2$ is divisible by 3. (25%)

d) Use the method of indirect proof, to prove that if n is an integer and $3n + 5$ is odd then n is even. (25%)

Q2 a). Let A, B and C be any three subsets of the universal set U. Use the laws of sets to prove that $(A - B) - (B - C) = A - B$. (20%)

b) Let the relation R in $\mathbb{N} \times \mathbb{N}$ is defined as $(a, b)R(c, d)$ if and only if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation. (20%)

c) If $f: x \longrightarrow x^2 + 4$, where $x \in (-\infty, 0)$, then show that f is a one-to-one function and find the inverse function of f. (30%)

d) Let $f: x \longrightarrow 2x^2$ for $x \in (0, 2)$ and $g: x \longrightarrow 2x - 1$ for $x \in (0, 1)$. Then find R_g , R_f and the function $g \circ f(x)$. (30%)

Q3 a) Let $A = \begin{bmatrix} 4a+1 & 4a \\ -4a & 1-4a \end{bmatrix}$.

i) Find the characteristic equation of A. (15%)

ii) Let $n \in \mathbb{Z}^+$ and $f(x)$ is a polynomial of degree $n-2$. If $x^n = (x-1)^2 f(x) + \lambda x + \mu$ then find the values of λ and μ in terms of n. (15%)

iii) Deduce that $A^n = nA + (1-n)I_2$. Hence prove that $A^n = \begin{bmatrix} 4an+1 & 4an \\ -4an & 1-4an \end{bmatrix}$, where I_2 is an identity matrix of order 2. (35%)

b) Apply elementary row (column) operations to find the factors of $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$. (35%)

Q4 a) Let λ be a real number such that $\lambda > 1$ and $(a_n)_{n\in\mathbb{Z}^+}$ be a sequence of positive numbers such that $a_1 > \sqrt{\lambda}$ and $a_{n+1} = \dfrac{\lambda(1+a_n)}{\lambda + a_n}$.

i) Express $(a_{n+1})^2 - 5$ in terms of a_n and use the principle of mathematical induction to prove that $a_n > \sqrt{5}$ for each n in $\mathbb{Z}^+$. (20%)

ii) Deduce that $(a_n)_{n\in Z^+}$ is a strictly decreasing sequence and find the $\lim\limits_{n\to\infty} a_n$. (15%)

b) Use D' Alembert's ratio test to show that $\sum\limits_{n=1}^{\infty} \dfrac{5^n + 2020}{3^n}$ is divergent. (20%)

c) Let f be a function defined as (25%)

$$f(x,y) = \begin{cases} \dfrac{x^2y^2}{8x^3+y^3}, & if\ (x,y) \neq (0,0) \\ 0 & if\ (x,y) = (0,0) \end{cases}.$$

Find $\lim\limits_{\substack{(x,y)\to(0,0)\\ y=mx}} f(x,y)$ and $\lim\limits_{\substack{(x,y)\to(0,0)\\ y=-2xe^x}} f(x,y)$, where $m \in \mathbb{R}$. What can you say about the $\lim\limits_{(x,y)\to(0,0)} f(x,y)$?

d) Show that $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0$ if $f(x,y) = e^x cosy$. (20%)

Q5 a) Let f be a function such that $f(x) = (x-4a)(x-7a)e^{-\frac{x}{2a}}$, where $a \in \mathbb{R}^+$.

i) Prove that $\dfrac{df(x)}{dx} = -\dfrac{1}{2a}(x-10a)(x-5a)e^{-\frac{x}{2a}}$. (15%)

ii) Verify Roller's theorem. (25%)

b) Let g be a function such that $g(x) = |x| sinx$. Show that g is continuous at $x = 0$ and g is differentiable at $x = 0$. (30%)

c) Let h be a function such that $h(x) = |x| cosx$. Show that h is continuous at $x = 0$ and h is not differentiable at $x = 0$. (30%)

Q6 a) Show that the differential equation

$$(6x^2 \cos y - 16y \cos x)dy + (12x\sin y + 8y^2 \sin x)dx = 7x^6$$ (40%)

is exact and find the general solution of the equation.

b) Use the substitution $y = vx$ to solve the following differential equation

$$10x \cos\left(\frac{9y}{x}\right)\frac{dy}{dx} = 10y \cos\left(\frac{9y}{x}\right) + 19x.$$ (30%)

c) Find the general solution of the following differential equation

$$\cos x\frac{dy}{dx} + y \sin x = 2021x^{2020} \cos^2 x.$$ (30%)

Q7. a). i) Prove that $\frac{1}{D+\alpha}f(x) = e^{-\alpha x}\frac{1}{D}e^{\alpha x}f(x).$ (20%)

ii) Use the formula in part i) to find the particular integral of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 5\sin 3x.$ (30%)

iii) Hence find the general solution of the differential equation in ii). (15%)

b) Let $y_P = ke^{2x}$ be a particular integral of the differential equation

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 30e^{2x}$. Then

i) find the value of k. (20%)

ii) find the general solution of the above differential equation. (15%)

Q8 A particle of mass m is projected with speed u along the greatest slope line of a plane inclined θ to the horizon. When the speed of the particle is v, the resistance on the particle is mkv^2, where $k > 0$.

a) Prove that the maximum distance travelled up by the particle is

$$\frac{1}{2k}\ln\left(1 + \frac{ku^2}{g\sin(\theta)}\right).$$ (30%)

b) If the speed of the particle is V when the particle returns to the point of projection, prove that $\frac{1}{V^2} = \frac{k}{g\sin(\theta)} + \frac{1}{u^2}.$ (50%)

c) Deduce that $V^2 < \frac{g}{k}\sin\theta$ and $u > V$. (20%)

End.

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