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THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Bachelor of Technology Honors in Engineering

Final Examination (2020/2021)
MPZ3552: Engineering Mathematics II

Date: 23rd January 2022 (Sunday) **Time: 14:00 – 17:00**

Instruction:

- **Please answer a total of five (05) questions choosing at least one from each single section.**
- **All symbols are in standard notation and state any assumption that you made.**

Important integrals

- $\int e^{ax} sinbx dx = \frac{e^{ax}}{a^2+b^2}(asinbx - bcosbx)$
- $\int e^{ax} cosbx dx = \frac{e^{ax}}{a^2+b^2}(acosbx + bsinbx)$
- $\int f(x)e^{ax}\,dx = \frac{1}{a}f(x)e^{ax} - \frac{1}{a^2}f^1(x)e^{ax} + \frac{1}{a^3}f^2(x)e^{ax}$ …… [Sign alternate $(+ - + - + \cdots)$]
- $\int f(x)cosax\,dx = \frac{1}{a}f(x)sinax + \frac{1}{a^2}f^1(x)cosax - \frac{1}{a^3}f^2(x)sinax - \cdots$ [Sign alternate in pairs $+ + - - + + - - \cdots$)]
- $\int f(x)sinax\,dx = -\frac{1}{a}f(x)cosax + \frac{1}{a^2}f^1(x)sinax + \frac{1}{a^3}f^2(x)cosax - \cdots$ [Sign alternate in pairs after the first term $(+ + - - + + - - + + - - \cdots)$]
- $\int secax\,dx = \frac{1}{a}ln|secax + tanax|$

SECTION – B

Q3.

I. Find the vector product and scalar product of the two vectors $3\underline{i} - 2\underline{j} + \underline{k}$ and $2\underline{i} - \underline{j} + 3\underline{k}$. [15%]

II. A particle moves in the XY plane such that its position vector at time t is given by $\underline{r} = 6(t-1)^2\underline{i} + (3t^4 - 4t + 2)\underline{j}$. Find the velocity and acceleration of the particle at a given time t? [10%]

III. Let $\underline{a}$, $\underline{b}$ and $\underline{c}$ are non-zero and non-parallel vectors. If $\underline{a} + 2\underline{b}$ parallel to $\underline{c}$ and $\underline{b} + 3\underline{c}$ parallel to $\underline{a}$, then show that $\underline{a} + 2\underline{b} + 6\underline{c} = 0$. [25%]

IV. Two particle A and B start to move simultaneously from point $A = (-5, 2)$ and $B = (3, -8)$ and moves with constant velocities $(3\underline{i} + \underline{j})$ and $(\underline{i} + \underline{j})$.

What will be the time and distance apart when they are closest together ? [20%]

V. Find the perpendicular distance from the point $2\underline{i} + 7\underline{j} + 5\underline{k}$ to the straight line joining two points $-4\underline{i} + 7\underline{j} + 8\underline{k}$ and $5\underline{i} + 2\underline{j} + 8\underline{k}$. [30%]

Q4.

I. Find the principle argument and argument of the following complex numbers. [20%]

a) $-1 - \sqrt{3}i$
b) $-3i$.

II. Let Z is a complex number and $\bar{Z}$ is complex conjugate of Z. Then find the values of Z and $\bar{Z}$ in the expression $4Z + 5\bar{Z} = \frac{17-4i}{1+i}$. [15%]

III. Let $Z \in \mathbb{C}$, then using the algebraic method, find the radius and center of the circle $|z+1| = 2|z-1|$ and sketch the graph of circle in Z plane. [15%]

IV. Let $ABCD$ is a quadrangle on an Argand diagram with the points $6 + 4i$, $10 + 3i$, $3 + 9i$ and $a + bi$. Two diagonals intersect at the point M and M divide both diagonal AC and BD in the ratio $1:2$ ($AM:MC = 1:2$ and $BM:MD = 1:2$). The point E is connected with point D as $|DE| = |DC|$ as well as angle between DE and DC is 90^0. Find the point E in the Argand diagram. [25%]

V. Using the De Moivre's theorem prove that [25%]

$$\cos 4\theta = 8cos^4\theta - 8cos^2\theta + 1.$$

Deduce that

$$sin4\theta = 2\ sin2\theta\ (1 - \ 2sin^2\theta).$$

SECTION – C

Q5.

I. Using the bisection method find the root of the function $f(x) = 10 - x^2$ correct to 4 decimal places in the interval $[-2, 5]$. [30%]

II. With the initial guess $x_0 = 1$, Use the Newton - Raphson method to find the value of the square root of 11 correct to eight decimal places. [30%]

III. Apply the Jacobi method to solve the following system. [40%]

$$5x_1 - 2x_2 + 3x_3 = -1$$
$$-3x_1 + 9x_2 + x_3 = 2$$
$$2x_1 - x_2 - 7x_3 = 3\ .$$

You may use the initial guess as $x_1 = x_2 = x_3 = 0$.
(Continue up to iterations)

Q6.

I. Construct a polynomial for the values given in following table, using Lagrange's interpolation formula and estimate $f(7)$. [25%]

x	5	6	9
y	12	13	14

II. The value of the function $y = f(x)$ at some discrete values are given in following table.

x	1891	1901	1911	1921	1931
$f(x)$	46	66	81	93	101

Find the value of $f(1895)$ by using. [40%]

a) Newton's forward difference interpolation formula, and
b) Newton's backward difference interpolation formula.

III. Evaluate $\int_0^{\pi} sin^2x\ dx$ by using Trapezoidal and Simpson's rules. Use 6 subintervals for the calculation. [35%]

SECTION – D

Q7.

I. Find the convergence interval of the following series: **[20%]**

a) $\sum_{n=0}^{\infty}\left(\frac{x}{5}\right)^n$

b) $\sum_{n=0}^{\infty}(x-7)^n$.

II. Find the power series for $f(x) = \ln(1+x)$. **[15%]**

III. Define the McLaurin's series expansion. **[35%]**

a) Find the McLaurin Series expansion of $f(x) = \cos x$ about $x = 0$.

b) Deduce the McLaurin Series expansion of $\sin 2x$ about $x = 0$.

IV. Define the Laplace transform for a function $f(t), 0 \leq t < \infty$. **[30%]**

Find the Laplace transformation of the following functions.

a) $\sin 3t + 5e^{-2t} + 2t^3$

b) $\sin(at+b) - at\cos at$.

Q8.

I. Let $f(x)$ be a periodic and Reimann integral function on the interval $(-\pi, \pi)$. The write down the Fourier series expansion for the function $f(x)$. **[10%]**

II. Let $f(x)$ be a function defined on the interval $(-5,5)$ by

$$f(x) = \begin{cases} 0, & -5 < x < 0 \\ 3, & 0 < x < 5. \end{cases}$$

Find the Fourier series expansion of $f(x)$. **[35%]**

III. Find the Fourier series reorientation of the function with the period 2π, defined by $h(x) = 2020x + 2021x^2 + 2022x^3$ for $-\pi < x < \pi$. **[55%]**

Hence deduce that $1 - \frac{1}{4} + \frac{1}{9} - \cdots = \frac{\pi^2}{12}$.

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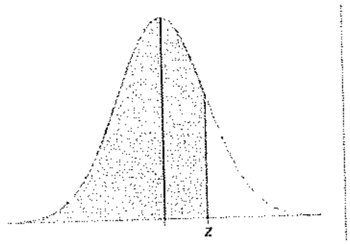
Standard Normal Cumulative Probability Table



Cumulative probabilities for POSITIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998