# THE OPEN UNIVERSITY OF SRI LANKA
**Faculty of Engineering Technology**
**Department of Mathematics & Philosophy of Engineering**

## Bachelor of Technology Honors in Engineering

**Final Examination (2020/2021)**
**MHZ3332: Engineering Mathematics IB**

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**Date: 23rd January 2022 (Sunday)** **Time: 14:00 pm – 17:00 pm**

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***Instructions:***

- *Answer* ***Five (05)*** *questions only.*
- *Number of pages in the paper is five (05).*
- *All the symbols are in standard notation unless they are defined.*

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**Important integrals**

- $\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$
- $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$
- $\int f(x)e^{ax}dx = \frac{1}{a}f(x)e^{ax} - \frac{1}{a^2}f^1(x)e^{ax} + \frac{1}{a^3}f^2(x)e^{ax} - \ldots\ldots\ldots$

  Sign alternate $(+-+-+-\ldots\ldots\ldots)$
- $\int f(x)\cos ax\,dx = \frac{1}{a}f(x)\sin ax + \frac{1}{a^2}f^1(x)\cos ax - \frac{1}{a^3}f^2(x)\sin ax\ldots\ldots\ldots$

  Sign alternate in pairs $(++--++--++--\ldots\ldots\ldots$
- $\int f(x)\sin ax\,dx = -\frac{1}{a}f(x)\cos ax + \frac{1}{a^2}f^1(x)\sin ax + \frac{1}{a^3}f^2(x)\cos ax\ldots\ldots\ldots$

  Sign alternate in pairs after the first term $(++--++--++--\ldots\ldots\ldots$

**Q1.**

I. Define the Fourier series expansion of $f(x)$ with period $2l$. Write down the formulae for the Fourier coefficients. (20%)

II. Let $f(x)$ be a function such that $f(x) = x^4, -\pi \le x \le \pi;\ f(x + 2\pi) = f(x)$ for all $x$.

a) Obtain the Fourier series expansion of the above function. (30 %)

b) Hence prove that

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4} = \frac{7\pi^4}{720}.$$

Hint: Use the result $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} = -\frac{\pi^2}{12}.$

(20%)

III. Determine the Fourier series for the function $f(t)$ defined on the interval $(-2, 2)$ given by

$$f(t) = \begin{cases} 0, & -2 < t < 0 \\ t, & 0 < t < 2 \end{cases};\ f(t + 4) = f(t).$$

(30%)

**Q2.**

I. Write down the Taylor series expansion of a function $f(x)$ about $x = a$. (10%)

II. If $y = \sin^{-1} 2x$,

a) Prove that

$$(1 - 4x^2)\frac{d^2y}{dx^2} = 4x\frac{dy}{dx}.$$

(15%)

b) Find the values of $\left(\frac{d^ny}{dx^n}\right)_{x=0}$ for $n = 1, 2, 3, 4$ and 5. (30%)

c) Hence, deduce the fifth order Taylor polynomial about $x = 0$, for $y = \sin^{-1} 2x$. (15%)

III.

a) Find the Taylor series expansion of $y = \ln t$ about $t = 1$. (15%)

b) Using the fourth order Taylor polynomial of $\ln t$ about $t = 1$, approximate the value of $\ln 1.5$. (15%)

**Q3.**

I. Using the method of variation of parameters, obtain a particular integral of the differential equation,

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^{4x}.$$

(30%)

II. Using a suitable trial function, Find a particular integral for the differential equation,

$$\frac{d^2y}{dx^2} - 4y = xe^x + \cos 2x.$$

Find the general solution of the above differential equation. (35%)

III. Using the result, $\frac{1}{D+\alpha}f(x) = e^{-\alpha x}\frac{1}{D}e^{\alpha x}f(x)$

Find a particular integral for the differential equation,

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 5(x+2).$$

Hence, obtain the general solution of the above differential equation. (35%)

**Q4.**

I. Define the Laplace transformation denoted by $L[f(t)] = F(s)$, of a function $f(t), 0 \le t < \infty$. (15%)

II. Find the Laplace transformation of the following functions.

a) $5e^{-6t} + 3\sin 2t - 9$ (15%)

b) $(t + 2e^{5t})\sin 3t$ (15%)

III. Find the inverse Laplace transformation of the following functions:

a) $\frac{10}{s^2+25} + \frac{4}{s-3}$ (15%)

b) $\frac{24e^{-7s}}{s^2-9}$ (15%)

IV. Using the Laplace transformation, solve the following differential equation with the initial conditions $y(0) = 1$ and $y'(0) = 2$.

$$\frac{d^2y}{dt^2} + y = 3\sin 2t.$$

(25%)

**Q5.**

I. The binomial distribution of a discrete random variable $X$ is define by

$$P[X = x] = \binom{n}{x} p^x (1-p)^{n-x} \text{ for } x = 0, 1, 2, \dots, n$$

where $n \in Z^+$ and $0 < p < 1$.

a) Derive the Poisson distribution as a limiting form of a binomial distribution. (20%)

b) Obtain the moment generating function of a Poisson distribution with parameter $\lambda$. (20%)

c) Hence find the mean and the variance of the distribution. (20%)

d) When packaging a product, a manufacturer finds that one packet in 25 is underweight. Determine the probabilities that in a box of 100 packets

$\alpha$) two (10%)

$\beta$) fewer than four will be underweight. (10%)

II. A machine produce nails whose length is normally distributed with mean $10\,cm$ and standard deviation $0.1\,cm$. It is required that the nails have specifications $10.05 \pm 0.12\,cm$. Any nail that does not meet above specification is regarded as faulty. What is the probability that nail produced by this machine be to correct specification? (20%)

**Q6.**

I.

a) Shade the area satisfying the following both inequalities.
$|z| \leq 6$ and $-6 \leq \text{Re}(z) + \text{Im}(z) \leq 6$. (10%)

b) Express $\text{Log}(-3)$ in the form of $x + iy$, where $x$ and $y$ are real numbers. (10%)

c) If, $f(z) = \dfrac{3z+1}{z-1}$, where $z \neq 1$.Show that $f$ is one to one. Also find the inverse function of $f$. (20%)

II. Let $\emptyset$ be a function of $x, y$ and $z$. A vector $\underline{\nabla\emptyset}$ is defined as follows.
$\underline{\nabla\emptyset} = \frac{\partial\emptyset}{\partial x}\boldsymbol{i} + \frac{\partial\emptyset}{\partial y}\boldsymbol{j} + \frac{\partial\emptyset}{\partial z}\boldsymbol{k}$. Prove that $\underline{\nabla}r^n = nr^{n-2}\boldsymbol{r}$ if $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ and $r = |\boldsymbol{r}| = \sqrt{x^2 + y^2 + z^2}$. (30%)

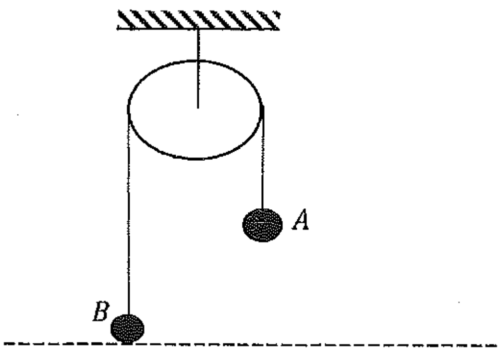
III. A particle $A$ starts to move with a constant velocity $5\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$ from the point $(1, 0, 2)$. Another particle $B$ starts to move at another time, moving in the same plane with the velocity $2\boldsymbol{i} - 3\boldsymbol{j} - \boldsymbol{k}$, passes through the point (2,7,6). Find whether $A$ and $B$ are collide. Neglect the air resistance and the masses of the particles. (30%)

**Q7.**

I. Two particles $A$ and $B$, each of mass $2m$ and $m$ respectively are attached to the two ends of a light inextensible string which passes over a fixed smooth weightless pulley. As shown in the figure, the particles are kept in rest with the particle $A$ hanging at a height $h$ from a horizontal floor, the particle $B$ touching the horizontal floor and the string being taut. Now in the same time, the system is released from rest and the particle $A$ is given an impulse $2mu$ vertically downwards direction.

a) Find the velocity of the particle $A$ just after the impulse. (20%)

b) Find the time taken by the particle $A$ to reach the floor. (20%)



II. A uniform rod $PQ$ mass $M$ and the length $8a$, which is free to rotate in a vertical plane about a smooth horizontal axis through $P$ is released from rest when $PQ$ horizontal. When the rod becomes vertical, a point $R$ of the rod where $PR = 6a$, strike on a fixed peg.

a) Find the moment of inertia of the rod about the axis through point $P$. (20%)

b) Applying the principle of energy conservation law, find the angular velocity of the rod just before the impact with the peg. (20%)

c) Hence, find the impulse exerted by the peg on the rod if the rod is brought to rest by the peg. (20%)

**Q8.**

Let $ABCD$ is a rectangular lamina with the side $AB = b$ and $BC = a$.
Find that the center of pressure of the rectangular lamina $ABCD$ if it is immersed in a liquid with the density $\rho$ such that the side $AB$ is on the surface of the liquid and the plane of the lamina is vertical.

(40%)

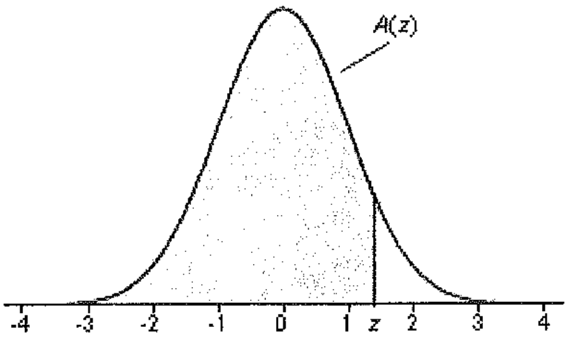
If the above lamina is immersed further such that the center of gravity of the lamina is at a depth $h$ $(h > \frac{2}{3}a)$ below to the surface of the liquid find the center of pressure of the rectangular lamina.

(60%)

*End*

## Z-table

### Cumulative Standardized Normal Distribution




$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to $z$ (in other words, the area under the curve to the left of $z$). It gives the probability of a normal random variable not being more than $z$ standard deviations above its mean. Values of $z$ of particular importance:

| $z$ | $A(z)$ | |
|---|---|---|
| 1.645 | 0.9500 | Lower limit of right 5% tail |
| 1.960 | 0.9750 | Lower limit of right 2.5% tail |
| 2.326 | 0.9900 | Lower limit of right 1% tail |
| 2.576 | 0.9950 | Lower limit of right 0.5% tail |
| 3.090 | 0.9990 | Lower limit of right 0.1% tail |
| 3.291 | 0.9995 | Lower limit of right 0.05% tail |

| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 | | | | | | | |