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THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Bachelor of Software Engineering Honors

Final Examination (2020/2021)
MHZ4256: Mathematics for Computing

Date: 15th January 2022 (Saturday) **Time: 14:00 – 17:00**

Instruction:

- **Answer only six questions.**
- **Please answer a total of six questions choosing at least one from each single section.**
- **All symbols are in standard notations. State any assumption that you made.**

SECTION – A

Q1.

I. Decide which of the followings are propositions and state the truth values of each proposition? [20%]

a) "If $9 - 15 \neq 7$ then, $12 + 3 = 17$ and $6 - 3 = 3$" ;

b) "$\forall x \in \mathbb{Z},\ |x| = x$ and $\forall x \in \mathbb{N},\ |x| = x$";

c) "Nimala is a beautiful girl in Bio-Maths class".

d) " The only odd prime number is 2 ".

II. State the "convers", "inverse", and "contrapositive" of each of the following statement: [30%]

a) If the science papers were not easy, then we do not pass the examination.

b) If $x + y$ is an irrational number, then x is an irrational number or y is an irrational number.

III. Let p, q and r be three statements.
Verify that $[(p \vee q) \rightarrow r] \leftrightarrow [\sim r \rightarrow \sim(p \vee q)]$ statement is a tautology or not. [10%]

IV. Show that $\sim(p \vee (\sim p \wedge q))$ and $\sim p \wedge \sim q$ are logically equivalent by using laws of the algebra of propositions. [20%]

V. Give the negation of the following statements: [20%]

a) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, [x + y = 2]$;

b) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, [x < y \Rightarrow x^2 < y^2]$.

Q2.

I. Test the validity of the following argument: [25%]

If there was a football match, then travelling was difficult.
If they arrived on time, then traveling was not difficult.
They arrived on time.

Therefore, there was no football match.

II. By using truth tables, prove distributive laws of propositions. [30%]

III. Using Mathematical induction, for a positive integer n, prove of the following:

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2 \text{ for all } n \geq 1.$$ [25%]

IV. Prove that $5x^2 + x + 7$ is odd for all integer x. [20%]

Q3.

I. Consider the following truth table. [25%]

A	B	C	L
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

Where A, B and C are input variables and L is an output.

a) Create a Karnaugh map (K-map) based on the above truth table.

b) Using the K-map created in the above part I.(a), write down the Boolean function.

II. Consider the following Boolean function given below. [30%]

$$F = ABC + A\overline{B}C + \overline{A}BC + \overline{A}\,B\,\overline{C}$$

Create a K-map based on the above Boolean function and minimized the above Boolean function using the K-map.

III. Minimize the following Boolean function by Algebraic method. [45%]

a) $F_1 = A\overline{B} + (B + \overline{B}C) + AB$

b) $F_2 = \overline{A}(A + BC) + (AC + \overline{B}C)$

c) $F_3 = AB + (A + B)(\overline{A} + B)$.

SECTION – B

Q4.

I. Write down the elements in each of the following set: [20%]

a) $A = \{x : x = 2n + 1,\ n \in \mathbb{Z}^+,\ n < 8\}$,

b) $B = \{x: x = n^3 + n^2, 0 \leq n < 5, n \in \mathbb{Z}\}$,

c) $C = \{x: x = 1 + (-1)^n,\ n \in \mathbb{Z}\}$;

d) $D = \{x :\ |x - 2| \leq 6, and\ x \in \mathbb{Z}^-\}$.

II. Let $L = \{1,2,3,4,5\}$, $M = \{x: x \in \mathbb{N}, |x - 4| < 7\}$, and $N = \{x: x^2 - 16 = 0, x \epsilon \mathbb{N}\}$.
With usual notation, find the element of following sets: [20%]

a) $(L\backslash M) \cup (M\backslash L)$;

b) $(L\backslash N) \cup (N\backslash L)$

III.

a) Define the Cartesian product of two sets. [05%]

b) Let $P = \{9, 99, 999\}$ and $Q = \{5, 55, 555\}$. Find $P \times Q$ and P^2. [20%]

IV. Let $E = \{a,\ b, \{a, b\}\}$. Find the power set $P(E)$ of E. [15%]

V. Without using Venn diagram, Show that

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. [20%]

Q5.

I. Find the Domain of the following functions:

a) $R_1(x) = \frac{x^3 - 3x + 1}{12x - 7}$, [05%]

b) $R_2(x) = \sqrt{x^4 - x^3 - 20x^2}$, [15%]

c) $R_3(x) = \sqrt{x - 1} + \sqrt{x + 6}$. [10%]

II. Let $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 3x - 1$ and $g(x) = \frac{x^2+1}{5}$. Give expressions for $f \circ f, f \circ g$ and $g \circ f$. [30%]

III. Let $A = \mathbb{R} - \{5\}$ and $B = \mathbb{R} - \{2\}$. Defined $h(x) = \frac{2x+3}{x-5}$. Prove that $h(x)$ is invertible and find a formula for $h^{-1}(x)$. [40%]

Q6.

I. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 2, 3, 4, 5, 6, 7\}$. In each of the following, find all the pairs of $A \times B$ that belong to R_1 and R_2.

a) $R_1 = \{(x, y) | \, x + 2 < y; \; x \in A, y \in B\}$, [10%]

b) $R_2 = \{(x, y) | x$ is even number and $2x + y$ is odd number, $x \in A, y \in B\}$.

[10%]

II.

a) Define the equivalence relation by the usual notation. [10%]

b) Determine whether the following relation is equivalence relation or not.

Let A be a set if integers and R_3 be the relation of $A \times A$ defined by $(a, b)R_3(c, d)$ if $a + d = b + c$. [30%]

c) A binary operation $R_4: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$, is defined by $a\, R_4\, \mathrm{b} = a \cdot |b|, \; \forall a, b \in \mathbb{Z}$. Show that R_4 is not commutative but associative. [20%]

III. Show that "division" on the set of natural numbers is a partial order [20%]

SECTION – C

Q7.

I. Simplify the following complex expression into standard form of complex number. [30%]

a) $(4 - i) + i(1 + 5i)^2 + 6$,

b) $\frac{1+2i}{1-i}$,

c) $\frac{(1-3i)(3+4i)}{(3-2i)^2}$.

II. Solve the following complex equation: [20%]
$z^2 + 2z + 2 = 0.$

III. Suppose $Z_1 = -2 - 3i$, $Z_2 = 5 + 2i$ and $Z_3 = 4i$. Find the value of following complex number: [20%]

a) $(\overline{Z_1} - Z_2)\overline{Z_3}$

b) $(Z_1 + \overline{Z_2})Z_3$.

c) Find the modulus, principal argument and argument of the following complex numbers. [30%]

a) $z = 1 - \sqrt{3}i$,

b) $z = -\sqrt{6} - \sqrt{2}i$,

c) $z = 1 + 2i$

Q8.

I. Let $z \in \mathbb{C}$. Find all complex numbers which satisfy $z\overline{(z)} = 100$ and a +b =14. [20%]

II. Let $Z_1 = -3 - 5i$, $Z_2 = 4 + 3i$ and $Z_3 = 3i$. Find the value of following: [30%]

a). $\frac{|-2Z_1 - 3Z_2|}{|Z_3|}$,

b). $\frac{|3Z_1 + 2Z_2|}{|2Z_3|}$.

III. Let $z \in \mathbb{C}$. Let $\frac{Z-2}{Z+1} = 3i$. Find the value of the z. [15%]

IV. Simplify the following expressions: [20%]

a) $\frac{13i^{26} + 5i^{17}}{i+1}$,

b) $\frac{45i^{31} + 35i^{10}}{2i+1}$.

V. If $-1 + \sqrt{2}i$ is a root of the equation $z^2 + 2z + 3 = 0$, then find the other root of the same equation. [15%]

Q9.

I. ABCD is a square with center at the $(1,2)$. If A represents the complex number $4+5i$. Find of the following: [25%]

a) complex numbers which represent the other vertices.

b) lengths of a diagonal and a side.

II. Draw the locus of the following: [25%]

a) $\arg(2-3z) = {}^{\pi}/_{3}$.

b) $\arg\left(\frac{1-5z}{1+\sqrt{3}i}\right) = \frac{5\pi}{6}$.

III. Sketch the set $\{z \in \mathbb{C};\ 2|z-2| = |z-3|\}$ in the complex plane. [20%]

IV. Let $z \in \mathbb{C}$, Draw the locus of $|z-2| = |z-1-\sqrt{3}i|$ and $|z-3| = 3$ in the same diagram and find the complex number at the intersection point of the loci. [30%]

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