The Open University of Sri Lanka

Department of Electrical and Computer Engineering

Final Examination 2015/2016

ECX6234 – Digital Signal Processing

Time: 0930 -1230 Hrs. Date: 2016- 11 - 28

Answer FIVE questions

Select at least **3** *questions from section* **B**

SECTION A

1.

(a) The signal $x(t) = \sin \Omega t$ is converted into a discrete signal $y[n]$. The sampling frequency is Ω_s. Express this signal in the form $x[n] = \sin[\omega_0 n]$ and find the values of $x[n]$ and ω_0 in terms of the parameters of the original continuous signal. [5 *marks*]

(b) Find whether the system described by the difference equation $y[n] = 2x[n] - x[n-1]$ is linear. [5 *marks*]

(c) Is the discrete system described by the input-output relationship $y[n] = n\, x[n]$ time-invariant? Justify your answer. [5 *marks*]

(d) What is a causal system? Give a difference equation which relates the input $x[n]$ with the output $y[n]$ of a discrete causal system. [5 *marks*]

2.

(a) (i) What is a BIBO stable system? [3 *marks*]

(ii) The impulse response of a linear time invariant (LTI) system is $h[n]$. If the input to the system is $x[n]$ and the output is $y[n]$

1.write the relationship between $x[n], h[n]$ and $y[n]$. [2 *marks*]

2.using the answer to (ii)1., show that the system is BIBO stable if $\sum_{k=-\infty}^{\infty} |h[k]| < \infty$.

[7 *marks*]

(b) The impulse response $h[n]$ of an LTI system is given by $h[n] = \alpha^n u[n]$, where α is a positive real number which lies between 0 and 1. Show that the system is BIBO stable. [8 *marks*]

3.

(a) The z-transform of $x[n]$ is given by $X(z) = Z\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$. Show that

$$Z\{x[n-L]\} = z^{-L} X(z)$$ [5 *marks*]

(b) Find the z-transform and the region of convergence (*ROC*) for $a^n u[n]$ from first principles, where a is a constant. [5 *marks*]

(c) Find the inverse z-transform of $X(z) = \frac{1}{z-3}, \quad |z| > 3$ [5 *marks*]

(d) A digital filter can be expressed by the input-output relationship

$$y[n] + a_1 y[n-1] + a_2 y[n-2] + ... + a_N y[n-N] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] +$$
$$..... + b_N x[n-M]$$

Derive an expression for the transfer function $H(z)$. [5 *marks*]

SECTION B

4.

(a)

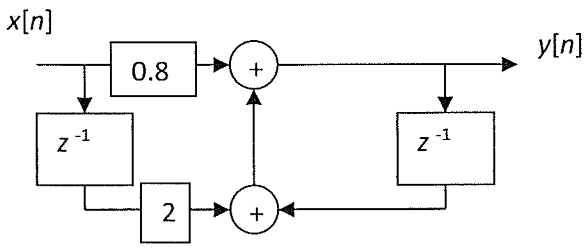

Fig.Q4

For the digital filter shown in Fig.Q4

(i) write state space equations. [5 *marks*]

(ii) find the frequency response $H(\omega)$. [5 *marks*]

(iii) find whether the filter is a FIR filter or an IIR filter. Justify your answer. [5 *marks*]

(b) Using the step response of the system, how can we find out whether the system described in (a) is BIBO stable? [5 *marks*]

5.

(a) A lowpass filter has a frequency response $h_L[n]$. Show that the filter having the impulse response $h[n] = (-1)^n h_L[n]$

(i) has the frequency response $H(\omega) = H_L(\omega - \pi)$, where $H_L(\omega)$ is the frequency response of the lowpass filter. [5 *marks*]

(ii) is a highpass filter. [5 *marks*]

(b) A reconstructor converts a discrete signal $x[n]$ into the analogue signal $x(t)$ using a zero order interpolating function $g(t)$. If the time between two consecutive values of $x[n]$ is T_S ($= \frac{1}{F_s}$),

(i) write an expression for $x(t)$ in terms of $g(t)$ and $x[n]$. [5 *marks*]

(ii) show that $X(F) = G(F)\ X(\omega)$.

$X(F)$ and $G(F)$ are the Fourier transforms of $x(t)$ and $g(t)$ respectively. $X(\omega)$ is the discrete time Fourier transform (DTFT) of $x[n]$. [5 *marks*]

6.

(a) N-point Moving Average Filter can be expressed by the input-output relation

$$y[n] = \frac{1}{N}\sum_{k=0}^{N-1} x[n-k]$$

(i) Find the impulse response of the filter. Is this a FIR filter? [5 *marks*]

(ii) Calculate and sketch the step response of the filter. [5 *marks*]

(b) An accumulator adds all the previous inputs to the present input.

(i) Write a difference equation to describe the input-output relationship of an accumulator. [5 *marks*]

(ii) Find the transfer function $H(z)$ of the accumulator. [5 *marks*]

7.

(a) Give basic steps to be followed (with reasons) when designing a FIR lowpass filter using window functions. [5 *marks*]

(b) A digital lowpass filter has a passband frequency of 6 kHz, stopband frequency 7 kHz with at least 40 dB attenuation and sampling frequency 42 kHz.

(i) Select a suitable window function to satisfy the requirements. [4 *marks*]

(ii) Find the length *N* of the filter. [4 *marks*]

(iii) Design a lowpass filter and derive an expression for the impulse response $h[n]$ of the filter. [7 *marks*]

8.

(a) What is *zero padding*? With the help of an example show how it improves some characteristics of the signal. [6 *marks*]

(b) Explain the orthogonality principle as applicable to minimization of mean-squared error. [6 *marks*]

(c) Give the estimator model of Kalman filter in block diagram form. Explain the function of each block. [8 *marks*]

Typical Window functions

Window type	$w(n)$	Δω	*Attenuation*
Rectangular	1	$\frac{4\pi}{N}$	-13dB
Bartlett	$\frac{2}{N-1}\left(\frac{N-1}{2}-\left\|n-\frac{N-1}{2}\right\|\right)$	$\frac{8\pi}{N}$	-27dB
Hanning	$0.5+0.5\cos\left(\frac{2\pi n}{N-1}\right)$	$\frac{8\pi}{N}$	-32dB
Hamming	$0.54+0.46\cos\left(\frac{2\pi n}{N-1}\right)$	$\frac{8\pi}{N}$	-43dB
Blackman	$0.42+0.5\cos\left(\frac{2\pi n}{N-1}\right)+0.08\cos\left(\frac{2\pi n}{N-1}\right)$	$\frac{12\pi}{N}$	-53dB