THE OPEN UNIVERSITY OF SRI LANKA
BACHELOR OF SCIENCE DEGREE PROGRAMME
FINAL EXAMINATION - 2021/2022
PHU 5313 - ADVANCED ELECTROMAGNETISM
Duration: TWO (02) HOURS

Date: 08th October 2022 **Time 1.30 pm – 3.30 pm**

$C = 2.99\times10^{8}\ ms^{-1}$, $\varepsilon_0 = 8.85\times 10^{-12}\ Fm^{-1}$, $\mu_0 = 4\pi\times 10^{-7}\ Hm^{-1}$

Answer Four (04) questions only

1. (a) Write down an example each for the vector product and scalar product that express one of the principles in electromagnetism mathematically.

   (b) Given that $\mathbf{A} = 2\mathbf{a}_x + 4\mathbf{a}_y$ and $\mathbf{B} = 6\mathbf{a}_y - 4\mathbf{a}_z$, find the angle between the two vectors by using the (i) vector product and the (ii) scalar product.

   (c) For three vectors **A**, **B** and **C**, given that $\mathbf{C} = \mathbf{A} - \mathbf{B}$. Show that, $C^2 = A^2 + B^2 - 2AB\cos\theta$, where θ is the angle between the vectors **A** and **B**.

   (d) Prove that $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}.\mathbf{C}) - \mathbf{C}(\mathbf{A}.\mathbf{B})$ for the three vectors **A, B** and **C**.

   (e) Use the above relation in (d) to show that

   (i) $\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B}.\nabla)\mathbf{A} - (\mathbf{A}.\nabla)\mathbf{B} + \mathbf{A}(\nabla.\mathbf{B}) - \mathbf{B}(\nabla.\mathbf{A})$ and

   (ii) $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla.\mathbf{A}) - \nabla^2\mathbf{A}$

   where **A** and **B** are two vectors and other symbols have their usual meaning.

2. (a) What is meant by (i) solenoidal and (ii) irrotational in the context of a vector field. Write down mathematically the conditions for a vector field to be solenoidal and irrotational.

   (b) Show that the vector field given by $\boldsymbol{r} = \mathrm{yz}\,\hat{\boldsymbol{r}}_x + \mathrm{xz}\,\hat{\boldsymbol{r}}_y + \mathrm{xy}\,\hat{\boldsymbol{r}}_z$ is both solenoidal and irrotational.

   (c) State the Helmholtz theorem and discuss its significance.

   (d) Hence show that four Maxwell's equations in electromagnetism under static state conditions are sufficient to describe electric and magnetic fields.

   (e) Suppose the gradient of a scalar field ϕ and curl of a vector field **A** are both the same vector, $\boldsymbol{r} = \mathrm{yz}\,\hat{\boldsymbol{r}}_x + \mathrm{xz}\,\hat{\boldsymbol{r}}_y + \mathrm{xy}\,\hat{\boldsymbol{r}}_z$ given in (b). Find the scalar field ϕ and vector field **A**.

3. (a) Explain what is meant by an equipotential surface by discussing its characteristics.

(b) Write down the uniqueness theorem and state the two boundary conditions that can be specified on a surface.

(c) A point charge $+Q$ is placed at a distance d away from the center of a grounded hollow conducting sphere of radius r, where $d > r$. Find the charge, q induced on the surface of the sphere by using the method of images.

(d) Hence show that the distance to the image charge q is, $a = \frac{r^2}{d}$ from the center of the above sphere in (c) along the line joining charge Q.

(e) Suppose the sphere described in (c) is disconnected from the ground and some external charge is placed on the surface to raise its potential to 10V. Estimate the charge placed on the sphere given that the radius of the sphere is 5 cm.

4. (a) Prove the following behavioral patterns of a conductor placed in an electric field using related equations where the symbols have their usual meaning

(i) $\mathbf{E} = 0$ inside a conductor

(ii) $\rho = 0$ inside a conductor

(iii) $\mathbf{E}$ just outside is σ/ε and perpendicular to the surface.

(b) Show that the capacity of a parallel plate capacitor is increased by a factor of $\frac{t}{t-d}$ if a conducting sheet of thickness d is introduced between the plates separated by a distance t.

(c) How do you define the electric displacement vector $\mathbf{D}$ for a static electric field?

(d) Derive the following relations by using the Gauss's theorem and divergence theorem where symbols have their usual meaning

(i) $\mathbf{D} = \varepsilon\, \mathbf{E}$ and

(ii) $\nabla . \boldsymbol{D} = \rho$

(e) The average electric field measured close to the surface of the earth is about 100 V that is induced due to a thunderstorm.

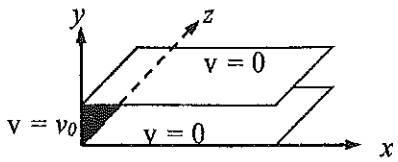
(i) What is the average surface charge density on earth?

(ii) Calculate the total charge on earth assuming that the earth is a conducting sphere of radius 6500 km.

5. (a) Write down the Laplace equation in a rectangular coordinate system for an electrostatic potential $\phi(x,y,z)$.

(b) If the potential is in the variable separable form, illustrate that, $\frac{1}{X}\frac{d^2X}{dx^2} = c_1$, $\frac{1}{Y}\frac{d^2Y}{dy^2} = c_2$, and $\frac{1}{Z}\frac{d^2Z}{dz^2} = c_3$ by substituting ϕ in the Laplace equation, where c_1, c_2, and c_3 are constants.

(c) Two grounded infinite metal plates lie parallel to xz plane, one at y = 0 and other at y = a. The left end at x = 0 is closed with an infinitely long metal strip at potential v_0 as shown in the figure. Write down the boundary conditions of this problem.



(d) Show that c_3 is equal to zero by analyzing the problem.

(e) Evaluate the potential at any point between the two plates by solving the Laplace equation.

6. (a) Write down Maxwell's equations for time varying electric and magnetic fields.

(b) Interpret each equation relating to the corresponding law in electromagnetism.

(c) Derive Maxwell's equation for free space from the equations of time varying electric and magnetic fields.

(d) Using Maxwell's equations in free space show that the electric and magnetic fields can be transmitted through free space with velocity $v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$.

You may use the relations, $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla.\mathbf{A}) - \nabla^2\mathbf{A}$ and $\frac{\partial^2 X}{\partial x^2} = \frac{1}{v^2}\frac{\partial X}{\partial t}$, where the symbols have their usual meaning.

(e) An electromagnetic wave of 100 MHz propagates in ferrite medium with relative permittivity of 10 and relative permeability of 100. Calculate the propagation velocity and the wavelength of the electromagnetic wave.

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