The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc. / B. Ed. Degree Programme

Department	**: Mathematics**
Level	**: 05**
Name of the Examination	**: Final Examination**
Course Title and - Code	**: Linear Programming – ADU5300**
Academic Year	**: 2021/22**
Date	**: 30.10.2022**
Time	**: 01.30 p.m. –03.30 p.m.**
Duration	**: Two Hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **(6)** questions in **(4)** pages.
3. Answer any **(4)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Involvement in any activity that is considered as an exam offense will lead to punishment.
6. Use blue or black ink to answer the questions.
7. Clearly state your index number in your answer script.

Answer any **FOUR** questions only

1. a) i) Write the definition of a ***convex set***. (05 marks)

   ii) Show that $S = \{(x_1, x_2, x_3): 2x_1 - x_2 + x_3 \leq 4\} \subset \mathbb{R}^3$ is a convex set. (10 marks)

   b) The advertising alternatives for a company include television, radio, and newspaper advertisements. The cost per television, radio, and newspaper advertisement is Rs. 20000, Rs.6000 and Rs 3000, respectively. The estimates for audience coverage per television advertisement is 100000, audience coverage per newspaper advertisement is 40000 and audience coverage per radio advertisement is 18000. The local newspaper limits the number of weekly advertisements from a single company to ten. Moreover, in order to balance the advertising among the three types of media, no more than half of the total number of advertisements should occur on the radio, and at least 10% should occur on television. The weekly advertising budget is Rs.182000.
   Formulate a linear programming model to maximize the total audience by clearly defining the decision variables. Do not solve. (10 marks)

   [Total marks 25]

2. a) i) Define the ***degeneracy*** in linear programming. (03 marks)

   ii) Consider the following initial table of a Simplex method applied to a maximization problem.

Basic variables	x_1	x_2	s_1	s_2	**RHS value**
s_1	1	4	1	0	8
s_2	1	2	0	1	4
z	-3	-9	0	0	0

   Write down the next iteration and obtain the optimal solution by avoiding cycling. (08 marks)

   b) Solve the following linear programming problem using the **revised simplex** method:

$$\begin{aligned} &\textit{Minimize } z = -6x_1 - 5x_2 \\ &s.t.\ \ 4x_1 + x_2 \leq 800 \\ &\quad 2x_1 + 3x_2 \leq 900 \\ &\quad x_1 \leq 180 \\ &\quad x_1, x_2 \geq 0 \end{aligned}$$

   (14 marks)

   [Total marks 25]

3. Consider the following linear programming problem:

$$\begin{aligned} &\textit{Maximize } z = x_1 + x_2 \\ &\textit{s.t. } x_1 - x_2 - x_3 = 1 \\ &\quad -x_1 + x_2 + 2x_3 - x_4 = 1 \\ &\quad x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

i) Solve the given problem using the Big- M method. (15 marks)

ii) Solve the given problem using the Two-phase method.
Explain the advantage of using the Two-phase method over to Big-M method for the given problem. (10 marks)

[Total marks 25]

4. a) i) Write the dual problem of the following primal problem:

$$\begin{aligned} &\textit{Minimize } z = 12x_1 + 14x_2 + 13x_3 \\ &\textit{s.t. } x_1 + 2x_2 + x_3 \leq 80 \\ &\quad 2x_1 + 2x_2 + 4x_3 \leq 40 \\ &\quad x_1 \leq 0,\ x_2 \geq 0,\ x_3 \leq 0 \end{aligned}$$

(05 marks)

ii) Solve the above constructed dual problem using iso-profit line method. (05 marks)

b) Solve the following linear programming problem using the **dual simplex method**:

$$\begin{aligned} &\textit{Maximize } z = -2x_1 - x_3 \\ &\textit{s.t. } x_1 + x_2 - x_3 \geq 5 \\ &\quad x_1 - 2x_2 + 4x_3 \geq 8 \\ &\quad x_1, x_2, x_3 \geq 0 \end{aligned}$$

(15 marks)

[Total marks 25]

5. A computer company has five expert programmers and the company needs to develop five application programs. The head of the computer company estimates the computer time (in minutes) required by the respective experts to develop the application programs as follows:

Program / Programmer	I	II	III	IV	V
P1	160	130	175	190	200
P2	135	120	130	160	175
P3	140	110	155	170	185
P4	50	50	80	80	110
P5	55	35	70	80	105

i) Formulate the model by clearly defining the decision variables. (05 marks)

ii) Use **Hungarian Algorithm** to find the assignment pattern that minimizes the time required to develop the application programs. (15 marks)

iii) Determine the programmers assigned to each application program. (03marks)

iv) Find the minimum time required to develop the application programs. (02 marks)

[Total marks 25]

6. A company has three factories, F1, F2, and F3 and four warehouses, W1,W2, W3 and W4. The shipping costs (in Rs) from factories to the warehouses are shown below in the form of a matrix:

	W1	**W2**	**W3**	**W4**	**Factory capacity in units**
F1	19	30	50	10	**7**
F2	70	30	40	60	**9**
F3	40	8	70	20	**18**
Warehouse requirement in units	**5**	**8**	7	**14**	

i) Using the information given in the above table, formulate a mathematical model in order to minimize the transportation cost.

(05 marks)

ii) Find the initial basic feasible solution (IBFS) using **North-West** corner method. (05 marks)

iii) Find the IBFS using **Vogel's Approximation** method.

(05 marks)

iv) Using the better IBFS obtained from ii) and iii), apply the **transportation algorithm** to show that the optimal transportation cost is Rs. 743.

(10 marks)

[Total marks 25]

********* END OF QUESTION PAPER *********