The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc. / B. Ed Degree Programme

Department	**: Mathematics**
Level	**: 05**
Name of the Examination	**: Final Examination**
Course Title and - Code	**: Numerical Methods – ADU5307**
Academic Year	**: 2021/22**
Date	**: 14.10.2022**
Time	**: 2.00 p.m. To 4.00 p.m.**
Duration	**: Two Hours.**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of (6) questions in (2) pages.
3. Answer any (4) questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Involvement in any activity that is considered as an exam offense will lead to punishment
6. Use blue or black ink to answer the questions.
7. Clearly state your index number in your answer script

1. (a) Using Newton-Raphson method, show that the iteration formula for finding the p^{th} root of $1/a$ is given by

$$x_{n+1} = \frac{(p-1)\,x_n^p + \frac{1}{a}}{p x_n^{p-1}}$$ where a is a real number and $n = 0, 1, 2, 3, \ldots\ldots$

Hence find $1/\sqrt{10}$ correct to four decimal places taking $x_0 = 0.5$.

(b) Show that for the equation $x^3 + x^2 - 1 = 0$, there exists a root in the interval (0, 1). Show that Simple Iteration method can be applied to find the root. Hence, find the root correct to four decimal places, by taking $x_0 = 1$.

2. (a) Prove that

(i) $E = \Delta + I$,

(ii) $E = (I - \nabla)^{-1}$,

(iii) $\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$

where Δ, ∇, E, I and δ are the forward difference, backward difference, the shift, identity and central difference operators respectively.

(b) The population of a town is given below

Year (x)	1911	1921	1931	1941	1951
Population in thousands (y)	15	20	27	39	52

(i) Using Gauss's backward interpolation formula find the population in 1926.

(ii) Using Gauss's forward interpolation formula find the population in 1936.

3. State and Prove the Trapezoidal rule.

Using the following data, evaluate $\int_1^7 f(x)dx$ by Trapezoidal rule taking $h = 3, 2$ and 1.

x	1	2	3	4	5	6	7
$f(x)$	2.105	2.808	3.614	4.604	5.857	7.451	9.467

Denote these integrals respectively by I_1 , I_2 and I_3 and applying Romberg's method for I_1, I_2 and I_2 , I_3 evaluate the integral.

4. (a) Applying Taylor series method of fourth order for the differential equation $\frac{dy}{dx} = x^2 - y$ subject to the initial condition $y(0) = 1$, evaluate $y(0.1)$ and $y(0.2)$ to four decimal places.

(b) Applying Taylor series method of the fourth order for the following differential equations

$$\frac{dy}{dx} = x + z,$$

$$\frac{dz}{dx} = x - y^2$$

subject to the initial conditions $y(0) = 2$ and $z(0) = 1$, evaluate $y(0.1)$ and $z(0.1)$ correct to four decimal places.

5. (a) Using Picard's method, find the first-three successive approximations to solve

$\frac{dy}{dx} = 1 + xy$ with the initial condition $y(0) = 1$.

(b) Applying Runge-Kutta method of fourth order, solve

$\frac{dy}{dx} = x^3 + \frac{y}{2}$ at $x = 1.1$, subject to the initial condition $y(1) = 2$.

6. Show that Milne's predictor -corrector formulae to solve the differential equation subject to the initial condition $y(x_0) = y_0$, be written as

$$y_{n+1\,p} = y_{n-3} + \frac{4h}{3}(2y'_{n-2} - y'_{n-1} + 2y'_n)$$

$$y_{n+1\,C} = y_{n-1} + \frac{h}{3}(y'_{n-1} + 4y'_{n-1} + 2y'_{n+1}).$$

Given $\frac{dy}{dx} = \frac{1}{x+y}$, $y(0) = 2$, $y(0.2) = 2.0933$, $y(0.4) = 2.1755$ and $y(0.6) = 2.2493$.

Find $y(0.8)$, by Milne's predictor-corrector.