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The Open University of Sri Lanka

B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME

Final Examination 2021/2022

Level 04 Applied Mathematics

ADU4300/ADE4300– Statistical Distribution Theory

**Duration: - Two hours**

**Date: - 22-10-2022** **Time: - 1.30 p.m. to 3.30 p.m.**

**Non programmable calculators are permitted. Statistical tables are provided.**

**Answer four questions only.**

1.

A company that produces a certain electrical product claims that the life time $\boldsymbol{X}$ (months) has the density function

$$f(x) = \frac{1}{10}e^{-\left[\frac{x-2}{10}\right]} \;\; ; \;\; x \geq 2$$

(i) Find the expected life time of a randomly selected electrical product.

(ii) Find the cumulative distribution function of the lifetime of the product.

(iii) Find the probability that a randomly selected product will not fail within 5 months.

(iv) Find the probability that a randomly selected product will fail within 5 months to 10 months.

(v) Find the probability that a randomly selected product will fail before 5 months or after 10 months.

(vi) Find the highest lifetime of the lowest 50% lifetimes of the product.

2.

(a) A box containing pins has a proportion $\boldsymbol{p}$ of the pins which are out of the specifications. A random sample of $\boldsymbol{n}$ pins was drawn with replacement. Suppose that $\boldsymbol{X}$ of them were out of the specifications. The probability mass function of $\boldsymbol{X}$ is given by

$$P_X\,(X = x) = n_{C_x} p^x (1-p)^{n-x}; \quad x = 0,1,2,3, \ldots\ldots\ldots.n$$

Let $M_x(t)$ be the moment generating function of X.

(i) Show that $M_x(t) = [\,1 + p(e^t - 1)]^n$

(ii) Using part (i), show that $E(X) = np$ and $Var(X) = np(1-p)$

(b) Batch that consists of 200 coil springs from a production process is checked for conformance to customer requirements. From the past experience, the mean number of nonconforming coil springs in a batch of 200 is 20. According to the past experience what is the probability that tested batch consists of more than 25 nonconforming coil springs.

(c) A restaurant kitchen has two food mixing machines ***A*** and ***B***. The average number of times that ***A*** brakes down per week is 0.4 and the average number of times that ***B*** brakes down per week is 0.1. Find the probability that no breakdowns of mixing machines ***A*** or ***B*** for a particular week. You may assume that breakdowns are immediately repaired and put back to work.

3.

(a)

Suppose that $X_1$, $X_2$, $X_3$, $X_4$ are independent random variables described as

$$X_1 \sim N(3,4) \quad X_2 \sim N(5,6) \quad X_3 \sim exp(3) \quad X_4 \sim gamma(2,3)$$

Find the following probabilities. Show your calculations and state the justifications clearly.

(i) $\Pr[\ 18 < (X_1 + 3X_2) < 25.62\ )]$

(ii) $\Pr[(X_3 + X_4) > 2\ )]$

(b)

Four white balls and three black balls are distributed in two urns in such a way that the first urn contains two white balls and a black ball. In a game, one ball is drawn randomly from the first urn and then places it in the second urn. Then a ball is drawn randomly from the second urn and places it in the first urn. This concludes the game. Let ***X*** denotes the number of white balls in the first urn.

(i) Find the possible values of ***X***

(ii) Calculate the expected number of white balls in the first urn

(iii) Calculate the ***E(4X+5)***

4.

(a)

In a quality control process, sample of 10 parts from a metal punching process are selected every hour. When the metal punching process is in control 1% of the parts require rework. Let ***X*** denotes the number of parts in the sample of size 10 that require rework.

The metal punching process is stopped for adjustments if $\mathbf{X > a + b}$

Where $\boldsymbol{a = E(X)\ when\ punching\ process\ is\ in\ control}$ and

$\boldsymbol{b = \sqrt{Var(X)}\ when\ punching\ process\ is\ in\ control}$.

(i) Suggest a suitable probability distribution for *X*? Clearly state the mass function and the parameters of the distribution.

(ii) Find the probability of punching process being stopped for adjustments when the process is in control.

(iii) Find the probability of process not being stopped for adjustments when 2% of the parts require rework.

(b)

For a Statistics course, total work load is distributed equally through 10 weeks and a student should spend an average of 12 hours studying per a week. Past experience gives the evidence to assume that the distribution of study hours per week of a student for the above course is normally distributed with a mean of 13 hours and a standard deviation of 2.6 hours. Assume that 150 students are registered to the course.

(i) How many students study 10 hours to 14 hours per week for the course?

(ii) Find the lowest study time of the highest 1% of the study times for the course per week.

(iii) Suppose that to get a pass grade for the course, a student should spend minimum of 10 hours per week. Find the expected number of students to be passed the course.

5.

A certain shop sells two brand of VCR *A* and *B*. Let *X* denote the number of brand *A* VCR machines sold per day and Y denote the number of brand *B* VCR machines sold per day. The following table shows the joint probabilities, according to the past data.

| ***P(x,y)*** | | ***x*** | | |
|---|---|---|---|---|
| | | 0 | 1 | 2 |
| ***y*** | 0 | 0.10 | 0.04 | 0.02 |
| | 1 | 0.30 | 0.14 | 0.06 |
| | 2 | 0.08 | 0.2 | ***k*** |

(i) Find the value of ***k***.

(ii) Find the marginal distribution function of ***X***.

(iii) What is the expected total number of sales of VCR on a randomly selected day?

(iv) What is the probability that on a randomly selected day, the number of brand *B* VCR machines sold is more than that of brand A?

(v) On a particular day salesman of the shop has sold their first brand *B* VCR at 10.00 a.m. Assume that the shop opens at 9.00 a.m. and closes at 5.00 p.m. What is the probability of no sales of brand *A* VCR on that day?

(vi) Find the conditional probability mass function of brand *A* VCR machines sold on a randomly selected day given that at least one brand *B* VCR machines is sold on that day.

(vii) Are the sales of brand A and brand B independent? Justify your answer.

6.

(a) A machine produces components of which mean and standard deviation of the diameter are 1.35 cm and 0.05 cm. The diameters are assumed to be normally distributed. Suppose all components with diameters outside the range 1.25 to 1.45 are considered as defective items and are rejected.

(i) What proportion of components will be rejected in a batch of production? Give your answer rounded to the second decimal place.

(ii) Find the probability of diameter of a randomly selected component is greater than 1.40 cm.

(iii) Suppose sample of 10 components are selected. Suggest a suitable probability distribution for sample mean. Clearly state the parameters of the distribution.

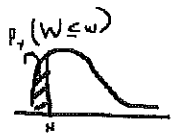
(iv) Find the probability that no defective items found in the sample.

(v) Suppose that production items will be checked one by one.

I. Find the probability that first defective item found is the 5[th] checked item.

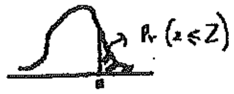
II. Find the probability that third defective item that found is the 9[th] checked item

**Left tail values of Standard Gamma Table**
**W - gamma(α,1)**
**This table contain the probabilities Pr(W ≤ w)**




| w | α = 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0.393469 | 0.264241 | 0.080301 | 0.018988 | 0.00366 | 0.000594 |
| 2 | 0.632121 | 0.593994 | 0.323324 | 0.142877 | 0.052653 | 0.016564 |
| 3 | 0.77687 | 0.800852 | 0.57681 | 0.352768 | 0.184737 | 0.083918 |
| 4 | 0.864665 | 0.908422 | 0.761897 | 0.56653 | 0.371163 | 0.21487 |
| 5 | 0.917915 | 0.959572 | 0.875348 | 0.734974 | 0.559507 | 0.384039 |
| 6 | 0.950213 | 0.982649 | 0.938031 | 0.848796 | 0.714943 | 0.55432 |

**Table of Standard Normal Probabilities**




**Let $Z \sim N(0,1)$. This table contains the probabilities $Pr(z \leq Z)$**

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 |
| 0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 |
| 0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 |
| 0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 |
| 0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |
| 0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| 0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| 0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2297 | 0.2266 | 0.2236 | 0.2207 | 0.2177 | 0.2148 |
| 0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| 0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 |
| 1 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| 1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| 1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1057 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| 1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| 1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 |
| 1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 |
| 1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| 1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 |
| 1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
| 1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 |
| 2 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 |
| 2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
| 2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
| 2.3 | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
| 2.4 | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0071 | 0.0069 | 0.0068 | 0.0066 | 0.0064 |
| 2.5 | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
| 2.6 | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 |
| 2.7 | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
| 2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| 2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| 3 | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |