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The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme

Department	**: Mathematics**
Level	**: 04**
Name of the Examination	**: Final Examination**
Course Title and - Code	**: Real Analysis 1 – PEU4300**
Academic Year	**: 2021/22**
Date	**: 13/10/2022**
Time	**: 1.30 p.m.-3.30 p.m.**
Duration	**: Two Hours.**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **(6)** questions in **(2)** pages.
3. Answer any 04 questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Draw fully labelled diagrams where necessary
6. Involvement in any activity that is considered as an exam offense will lead to punishment
7. Use blue or black ink to answer the questions.
8. Clearly state your index number in your answer script

(01) (a) Using the definition of limit, prove that $\lim_{n\to\infty} \frac{n+2}{n-\frac{3}{2}} = 1$.

(b) Using the definition of a sequence diverges to infinity,

prove that the sequence $\left\langle \frac{n^2+5n}{3n+1} \right\rangle$ diverges to infinity.

(c) Let $x_1 = 1$ and $x_n = \sqrt{2 + x_{n-1}}$ for each n≥ 2.

Show that $\langle x_n \rangle$ is bounded above by 2 and monotone.

(02) (a) State the sandwich theorem for limits of sequences.

Prove that $\left\langle 4 + \frac{\cos n}{3n} \right\rangle$ converges and find its limits.

(b) Suppose $\langle x_n \rangle$ and $\langle y_n \rangle$ are real sequences and l is a real number such that for each

$n \in \mathbb{N}$, $|x_n - l| \le y_n$ and $\lim_{n\to\infty} y_n = 0$, Prove that $\lim_{n\to\infty} x_n = l$.

Deduce that $\lim_{n\to\infty} \frac{3n^4+1}{n^4+n^2} = 3$.

(c) Write down the limsup and liminf of the sequence $\left\langle (-1)^n + \frac{1}{n} \right\rangle$.

(03) (a) Using the definition, show that the sequence $\langle 4 + (-1)^n \rangle$.

(b) Let $\langle x_n \rangle$ be a bounded divergent sequence of real numbers and let $\langle y_n \rangle$ be a

Sequence that converges to zero. Prove that the sequence $\langle x_n \, y_n \rangle$ converges to zero.

Deduce that $\left\langle -\left(\frac{1+(-1)^n}{2n}\right) \right\rangle$ converges to zero.

(04) (a) If $x_n = 1 + \frac{(-1)^n}{2n}$, find the least positive integer m such that $|x_n - 1| < \frac{1}{10^3}$

for each $n > m$.

(b) Discuss the boundness of the following sequence $\langle a_n \rangle$ is given by

$a_n = 1 + \frac{1}{3} + \frac{1}{3^2} + \cdots + \frac{1}{3^n}$ for each $n \in \mathbb{N}$.

(c) Discuss the convergence of the series $\sum_{n=1}^{\infty} (-1)^{n+1}$.

(d) Show that $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$ converges to $\frac{11}{18}$.

(05) Determine the convergence or divergence of each of the following series:

(i) $\sum_{n=1}^{\infty} \frac{1}{2n-1}$

(ii) $\sum_{n=1}^{\infty} \frac{n^n}{(n+1)^{n+1}}$

(iii) $\sum_{n=1}^{\infty} \frac{1}{2^n+3^n}$

(iv) $\sum_{n=1}^{\infty} \frac{2^n\, n!}{n^n}$

(v) $\sum_{n=1}^{\infty} \frac{(n-\ln n)^n}{2^n\, n^n}$

(06) (a) Define the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n\, x^n$.

Determine the radius of convergence of each of the following power series:

(i) $\sum_{n=1}^{\infty} \left(\frac{n+3}{n}\right)^n x^n$

(ii) $\sum_{n=0}^{\infty} \frac{n^n}{n!}\, x^n$

(b) Determine whether each of the following series is absolutely convergent or divergent.

(i) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \sin n\alpha}{n^2}$, α is real.

(ii) $\sum_{n=1}^{\infty} \frac{(-1)^n\, n^{100}}{2n!}$

(iii) $\sum_{n=1}^{\infty} \frac{-n+2}{n^3+1}$