The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc. / B. Ed Degree Programme

Department	**: Mathematics**
Level	**: 04**
Name of the Examination	**: Final Examination**
Course Title and - Code	**: Continuous Functions – PEU4315**
Academic Year	**: 2021/22**
Date	**: 26.10.2022**
Time	**: 01.30 p.m. To 03.30 p.m.**
Duration	**: Two Hours.**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of (6) questions in (2) pages.
3. Answer any (4) questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Draw fully labelled diagrams where necessary.
6. Involvement in any activity that is considered as an exam offense will lead to punishment.
7. Use blue or black ink to answer the questions.
8. Clearly state your index number in your answer script.

1. (a). State the definition of a limit point.

   (b). Let $E = [0, \infty)$ and $f: E \to \mathbb{R}$ be $f(x) = 3x - 5$, for all $x \in E$.

   Prove that $\lim_{x \to 2} f(x) = 1$.

   (c). Let $E = \{x_1, x_2, x_3, \dots, x_n | x_i \in \mathbb{R} \text{ for } i = 1,2, \dots, n\}$.

   Find the limit points of the subset E of $\mathbb{R}$, if they exists.

[25 Marks]

2. (a). State whether the following statements **A – C** are **correct** or **incorrect.** Justify your answer.

   A. The set $\{2020, \ 2021, 2022\}$ has a limit point.

   B. Let the function $f: E \to \mathbb{R}$ be given by $f(x) = x + 2$, where $E = \{2\} \cup [3, 4)$. Then we cannot discuss the limit of $f(x)$ as x tends to 2.

   C. The set $\left(3, \frac{7}{2}\right) \cup \left(\frac{7}{2}, 4\right)$ is a deleted ε – neighborhood of $\frac{7}{2}$ when $\varepsilon = \frac{1}{2}$.

   (b). Let $A = \left\{\frac{n+1}{2n+2}; n \in \mathbb{N}\right\}$. Is $\frac{1}{2}$ a limit point of A? Justify your answer.

[25 Marks]

3. (a). Let $f(x) = f(x) = \begin{cases} x & \text{if } x < 1 \\ 3 - x & \text{if } x \geq 1 \end{cases}$.

   (i). Find the value of $\lim_{x \to 1-} f(x)$. Justify your answer using the definition.

   (ii). Find the value of $\lim_{x \to 1+} f(x)$. Justify your answer using the definition.

   (iii). Does the $\lim_{x \to 1} f(x)$ exist? Justify your answer.

   (b). Let $h(x) = x^3 + x$ defined on the interval $(0, 2)$. Prove that $\lim_{x \to 1} h(x) = 2$.

[25 Marks]

4. (a). Let $\boldsymbol{f}, \boldsymbol{g}$ be functions, $\boldsymbol{c_1}, \boldsymbol{c_2}$ be real numbers such that $(\boldsymbol{c_1}, \infty) \subseteq \boldsymbol{Domn(f)}$, $(\boldsymbol{c_2}, \infty) \subseteq \boldsymbol{Domn(g)}$. Suppose that $\lim_{x \to \infty} \boldsymbol{f(x)}$ and that $\lim_{x \to \infty} \boldsymbol{g(x)}$ exist. Prove that $\lim_{x \to \infty} [\boldsymbol{f(x)} + \boldsymbol{g(x)}]$ exists and $\lim_{x \to \infty} [\boldsymbol{f(x)} + \boldsymbol{g(x)}] = \lim_{x \to \infty} \boldsymbol{f(x)} + \lim_{x \to \infty} \boldsymbol{g(x)}$.

   (b). Let $f(x) = \frac{2x^2+6x+3}{5x^2+3x}$ for each $x > 0$. Show that $\lim_{x \to \infty} f(x) = \frac{2}{5}$.

[25 Marks]

5. (a). (i). Suppose f is a function defined on an interval (a, b) and f is continuous at c where $c \in (a, b)$. Prove that $|f(x)|$ is continuous at c.

(ii). Is it true that $|f(x)|$ is always discontinuous at a point where the function $f(x)$ is discontinuous at the same point.

(b). Let $f(x) = \begin{cases} x^2 + 2, & x \geq 1 \\ 2x + 1, & x \leq 1 \end{cases}$.

Using definition prove that $f(x)$ is continuous at 1.

[25 Marks]

6. (a). State the definition of a continuous function on an open interval.

(b). Define $h: (-1,\ 5) \to \mathbb{R}$ by $h(x) = \sqrt{5 + 4x - x^2}$ for each $x \in (-1, 5)$. Show that h is continuous on $(-1, 5)$.

[25 Marks]

...End...