The Open University of Sri Lanka
Department of Mathematics
Advanced Certificate in Science Programme
MYF2520/MHF2520- Combined Mathematics 2 – Level 2
Final Examination 2021/22

Date: 21-01-2023 **From 9:30 am. To 12:30 pm.**

Answer **All** Questions in **Part A** and Answer **Five** Questions in **Part B.**

PART A

1. In the trapezium $ABCD$, DC and AB are parallel, and the length of the side DC is equal to twice that of AB. Show that $\overrightarrow{AB} + \overrightarrow{CB} + \overrightarrow{CD} = \overrightarrow{CA}$.

2. If $|\underline{a}| = |\underline{b}|$, then show that the vectors $(\underline{a} + \underline{b})$ and $(\underline{a} - \underline{b})$ are perpendicular to each other.

3. When two forces P and Q are acting inclined at an angle θ to each other, their resultant is $5\sqrt{P^2 + Q^2}$. When the two forces are inclined at $(90^0 - \theta)$, the resultant is $3\sqrt{P^2 + Q^2}$. Show that $\tan\theta = \frac{1}{3}$.

4. A particle of weight W is suspended by two strings attached to it. The other ends of the strings are attached to the same horizontal level. One string is inclined at 30° to the horizontal. What should be the direction of the other string for its tension to be least .

5. One end of a uniform rod is hinged to a vertical wall. It is kept in equilibrium by a string attached to the other end of the rod. The string is attached to a point on the same vertical wall directly above hinge. The rod and the string are both inclined at the same angle θ to the horizontal. If the weight of the rod is W, show that the reaction at the hinge is $\frac{W}{4}\sqrt{8 + cosec^2\theta}$.

6. A cyclist, travelling with constant acceleration along a straight road, passes three points A, B and C, where $AB = BC = 20\ m$. The speed of the cyclist at A is $8\ ms^{-1}$ and at B $12\ ms^{-1}$. Find the speed of the cyclist at C.

7. A ball is projected vertically upwards, from a point O, with initial velocity $32\ ms^{-1}$. Ignoring air resistance, find the time it takes the ball to return to O, under the gravitational force.

8. A projectile is launched at an angle 45° to the horizontal. It lands 1.28 km from the point of projection. Calculate the initial speed.

9. Two particles are connected by a light inextensible string which passes over a smooth fixed pully. The heavier particle is held so that the string is taut, and the parts of the string not in contact with the pully are vertical. When the system is released from rest the particles have an acceleration of $\frac{1}{2}g$. Find the ratio of the masses of the particles.

10. A girl is riding a bicycle along a straight path at 5kmh^{-1}. A boy is riding a bicycle 3 kmh^{-1} along a perpendicular straight path towards the path of the girl. What is the velocity of the boy relative to the girl?

PART B

11. (a) The diagonals of a quadrilateral $ABCD$ intersect at O. X and Y are the mid points of the diagonals AC and BD respectively.

Show that (i) $\overrightarrow{BA} + \overrightarrow{BC} = 2\overrightarrow{BX}$.

(ii) $\overrightarrow{BA} + \overrightarrow{BC} + \overrightarrow{DA} + \overrightarrow{DC} = 4\overrightarrow{XY}$.

(iii) $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} = 4\overrightarrow{OM}$

(b) $\underline{a}$ and $\underline{b}$ are two vectors such that $|\underline{a}| = 2$ and $|\underline{b}| = 3$. The angle between $\underline{a}$ and $\underline{b}$ is $\frac{2\pi}{3}$. Find $\underline{a}.\underline{b}$. Calculate $|\underline{a} + 2\underline{b}|$ and $|\underline{a} - 2\underline{b}|$ and $(\underline{a} + 2\underline{b}).(\underline{a} - 2\underline{b})$.

Hence, find the angle between $\underline{a} + 2\underline{b}$ and $\underline{a} - 2\underline{b}$.

12. In the rectangle $ABCD$. $AB = 8\ m$, and BC=6 m. P, Q, R and S are the mid points of the sides AB, BC, CD and DA respectively. Forces whose magnitudes are 5, 10, 15, 20, λ, and μ act along $\overrightarrow{PQ}, \overrightarrow{QR}, \overrightarrow{RS}, \overrightarrow{SP}, \overrightarrow{AC}$ and $\overrightarrow{BD}$ respectively.

    (a) Show that the system of forces is not in equilibrium.

    (b) If the System of forces reduces to a couple, show that $\lambda = \mu = 10$.

    (c)) If the System of forces reduces to a single force acting at C, show that $\mu = 35$.

13. A uniform circular plate has a radius a and weight W. One end of a light inextensible string of length $2a$ is attached to a point on the edge of the plate and the other end is attached to a fixed point on a smooth vertical wall in such a way that the pate is in equilibrium in a vertical plane perpendicular to the wall and touching it. Find the tension in the string and reaction on the plate from the wall.

14. As a car passes the point A on a straight road, its speed is $10\ ms^{-1}$. The car moves with constant acceleration $a\ ms^{-2}$ along the road for T seconds until it reaches the point B, where its speed is $V\ ms^{-1}$. The car travels at this speed for a further 10 seconds, and reaches the point C. From C it travels for a further T second with constant acceleration $3a\ ms^{-2}$ until it reaches a speed $20\ ms^{-1}$ at the point D. Sketch the (t, v) graph for the motion, and show that $V = 12.5\ ms^{-1}$.
    Given that the distance between A and D is 675 m, find the values of a and T.

15. A fielder can throw a cricket ball faster at low angles than at high angles. This is modelled by assuming that, at an angle θ, he can throw a ball with a speed $k\sqrt{\cos\theta}$ where k is a constant.

    (a) Show that the horizontal distance he can throw is given by $\frac{2k^2}{g}(sin\theta - sin^3\theta)$

    (b) Find the maximum distance he can throw the ball on level ground.

16. A van of mass 1200 kg is towing a car of mass 800 kg up a slope inclined at 8° to the horizontal. The resistance to the motion of the van may be modelled by a single force of magnitude 500 N acting parallel to the slope. For the car the resistance may be modelled by a single force of magnitude 200 N acting parallel to the slope. The van is travelling at constant speed. Stating one assumption that you have made, find

(a) the tension in the tow rope between the van and the car.

(b) the driving force acting on the van.

The driving force acting on the van is now increased to 4000 N. Find the time taken for the van to increase its speed from $10\ ms^{-1}$ to $14\ ms^{-1}$.

17. (a) A gun of mass M rests inclined at an angle α to the horizontal so that it can recoil freely on a smooth horizontal plane. A bullet of mass m is fired from it with a velocity v inclined at an angle θ to the horizontal. Show that $\tan\theta = \left(1 + \frac{m}{M}\right)\tan\alpha$.

(b) A sphere A of mass 1 kg moving with a velocity $10\ ms^{-1}$ collide directly with a sphere B of mass 2.5 kg moving in the same direction with a velocity of $8\ ms^{-1}$. If velocities of A and B after impact are u and v respectively and if the coefficient of restitution is e, show that $u = \frac{10}{7}(6 - e)$ and $v = \frac{4}{7}(15 + e)$.

Hence, deduce also that $\frac{50}{7} \leq u \leq \frac{60}{7}$ and $\frac{60}{7} \leq u \leq \frac{64}{7}$.