The Open University of Sri Lanka
Department of Mathematics
Advanced Certificate in Science Programme
MYF2522/MHF2522- Combined Mathematics 4 – Level 2

Final Examination 2021/22

Date: 28-01-2023 **From 9:30 am. To 12:30 pm.**

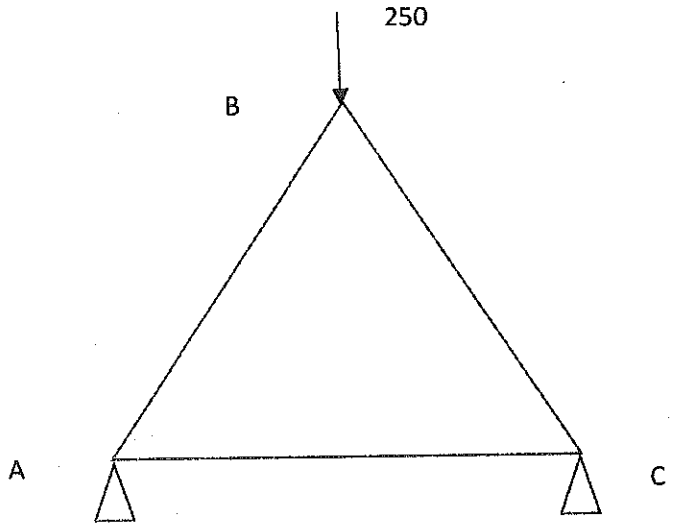
Answer **All** Questions in **Part A** and Answer **Five** Questions in **Part B.**

PART A

1. A Uniform ladder of weight W is in limiting equilibrium with one end touching a smooth vertical wall and foot of the ladder resting on a rough floor that is inclined at an angle α to the horizontal. Prove that the inclination of the ladder to the wall is $\tan^{-1}[2tan(\lambda - \alpha)]$, where λ is the angle of friction.

2. The weights of two equal uniform rods AB and AC smoothly hinged at A, are W each. A weight of W' is suspended at a point on AC at a distance of $\frac{3}{4}AC$ from A and the two rods are kept in equilibrium with B and C resting on a smooth horizontal plane by a light inextensible string connecting B and C. If $B\hat{A}C = 2\theta$, show that the tension in the string is $\left(\frac{4W+W'}{8}\right)\tan\theta$.

3. A framework consisting of three equal light rods is shown in the figure. It is held in equilibrium on two smooth supports at A and C and vertex B exerts a weight of 250N. Find the stress of the rods and determine whether they are tensions or thrusts. AC is horizontal.



4. The lengths of minutes hand and the seconds hand of a clock are 8cm, and 10cm respectively.

   (a) Find the angular velocity of the seconds hand in radians per second.
   (b) The distance marked by the end point of the minutes hand in an hour.

5. A particle P of mass m is at rest on the top of a fixed sphere with center O and radius a. It is suddenly displaced from its equilibrium position.

   Show that when the line OP makes an angle θ with the vertical, $\dot{\theta}^2 = \frac{2g}{a}(1 - \cos\theta)$.
   Find the reaction on the particle from the surface of the sphere.

6. Two particles A and B of mass M and m respectively are attached to the two ends of a light inextensible string of length $3l$ passing through a smooth ring. The particle B moves in a horizontal circle with centre A and the particle A is in equilibrium at a distance l vertically below the ring. Find the angular velocity of B and the mass of B in terms of M.

7. A car of mass 6000kg travels along a slope inclined at $\sin^{-1}\left(\frac{1}{15}\right)$ to the horizontal with a constant velocity of $5ms^{-1}$. If the power of the engine is 25kW, find the resistance to the motion of the car.

8. The shirt collar size of a class of advanced level students are given below.

x	Number of Students, f
15	3
15.5	17
16	29
16.5	34
17	12

   Find for these data

   (a) the mode, (b) the median, and (c) the mean.

9. Events A and B are independent and $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{5}$.
   Find

   (a) $P(A \cap B)$, (b) $P(A \cap B')$, (c) $P(A' \cap B')$.

10. For two events A and B, $P(A \cap B') = 0.32$, $P(A' \cap B) = 0.11$ and $P(A \cup B) = 0.65$.

    (a) Write down the value of $P(A)$ and the value of $P(B)$.
    (b) Find $P(A/B')$.
    (c) Determine whether or not A and B are independent.

PART B

11. Two equal uniform rods AB and BC of length $2a$ are jointed rigidly at B. $A\hat{B}C = \frac{\pi}{2}$. This system is in limiting equilibrium with AB forming a horizontal tangent and the rod BC forming a vertical tangent to a rough ring of radius r. Assuming that $a > r$ and the coefficient of friction between the rods and the ring is μ, show that $\frac{1-\mu}{1+\mu^2} = \frac{a}{2r}$.

12. The length of the rods AB and AC of weight W and smoothly jointed at A are $5m$ each. The ends B and C are placed on a smooth horizontal table so that $BC = 8m$. The mid points of AB and AC are D and E respectively. The system is kept in a vertical plane by two light inextensible strings CD and BE.

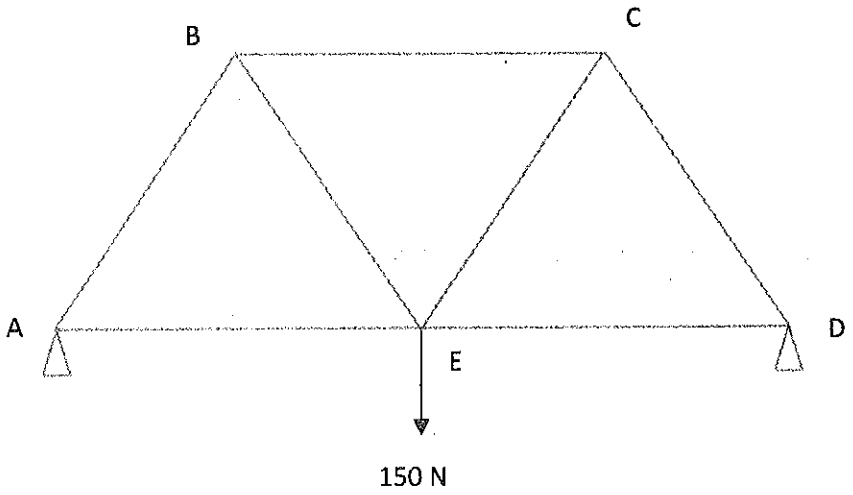
    (a) Show that $B\hat{C}D = tan^{-1}\frac{1}{4}$.
    (b) Find the tension in the strings and the reaction at A.

13. Show that the centre of mass of a uniform sector of radius r and subtending an angle 2α at the centre is at a distance $\frac{2r\sin\alpha}{3\alpha}$ on the symmetrical axis from the centre.

    Find the centre of mass of a sector AOB of radius r and semi vertical angle α. When this sector is suspended freely from the point A, its axis makes an angle θ with vertical.

    Show that $\tan\theta = \frac{3\alpha\tan\alpha}{3\alpha - 2\tan\alpha}$.

14. A framework of seven equal smoothly jointed light rods is shown in the figure. It rests in a vertical plane on two supports at A and D. A weight of 150 Nis suspended at E. Find the stresses of the rods and determine whether they are tensions or thrust AED is horizontal.



15. One end of a light elastic string of natural length a and modulus of elasticity $4mg$ is tied to a fixed-point O and the other end is attached to a particle of mass m. Let e be the extended length of the string, when the particle is in equilibrium.
Show that $e = {}^{a}/_{4}$.
The particle is pulled vertically downward a distance a below its equilibrium position and released. Show that a part of the motion of the particle is simple harmonic. Show also that height from the position of equilibrium to the maximum point it ascends during the motion is $\frac{17a}{8}$ and that the period of oscillation is $T = \sqrt{{}^{a}/_{g}}\left[\pi - \cos^{-1}\frac{1}{4} + \sqrt{15}\right]$.

16. The maximum velocities that can be acquired by a car of mass M on level ground is v. The resistance of the road against its motion is constant and the efficiency of the engine is H.

Find the maximum velocity the car can acquire when it travels straight up and down a slope inclined at an angle α to the horizontal, w

here $\sin\alpha = \frac{1}{k}$. If the velocity descend is twice the velocity of ascend, show that $3Mgv = kH$, show also that when this car is travelling with velocity of $\frac{v}{2}$ on level ground, its acceleration is $\frac{H}{Mv}$.

17. A screening test for a particular condition is not 100% reliable. In fact, the probability that a person who has the condition tests positive is 0.97. The probability that a person who does not have the condition tests positive is 0.01. The proportion of the population which has the condition is 1.5%.

A person is selected at random and tested for the condition.

(a) Using A for the event that the person does not have the condition and B for the event that the person tests positive, illustrate this situation on a tree diagram.

(b) Find the probability that the person tests positive.

(c) If the person tests positive, find the probability that the person does not have the condition.

(d) In light of your answer to part (c), comment briefly on the effectiveness of the test.