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The Open University of Sri Lanka
B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME
Open Book Test - 2021/2022
Level 05 - Applied Mathematics
ADU5305/ADE5305– Statistical Inference

Duration: - One hour.

DATE: - 14-01-2023 **Time: - 2.30 p.m. – 3.30 p.m.**

Non programmable calculators are permitted.

Answer all questions

1.

Suppose that the daily travel time to *ABC* school of a randomly selected student, follows a normal distribution. However, the mean travel time and variance travel time to the school is unknown. Suppose total number of students in the school is 800. Random sample of 20 students were selected and travel time to school was collected.

(i) What is the population of interest? Is the population finite? Justify your answer.

(ii) Derive moment estimators for the mean and the variance of daily travel time to school of a randomly selected student.

(iii) Daily travel time of 20 randomly students in minutes are given below. Using part (ii) estimate the mean and the variance of daily travel time to school of a randomly selected student.

94	70	43	28	62
11	27	66	34	46
66	79	66	74	20
98	58	60	73	69

(iv) Estimate the standard error of the estimated mean given by you in part (iii).

(v) Estimate the sample size required to estimate the mean daily travel time to school of a randomly selected student with an error bound of 5 minutes at 95% confidence.

Note: $Z_{0.025} = 1.96$ and $Z_{0.05} = 1.64$

2.

(a) Suppose that $\hat{\theta}$ is an estimator for parameter θ. State whether the following statements are true or false. In each case justify your answer.

(i) $\hat{\theta}$ is an unbiased estimator for parameter θ implies that $\hat{\theta}$ is a precise estimator for parameter θ.

(ii) $Var(\hat{\theta}) = \frac{\theta}{n}$ and $\hat{\theta}$ is an unbiased estimator for parameter θ implies that $\hat{\theta}$ is a consistent estimator for parameter θ.

(b) Let $X_1, X_2, X_3, \ldots\ldots\ldots X_n$ be a random sample from a uniform distribution with density given by

$$f(x;\theta) = \frac{1}{1-\theta} \quad ; \; 0 \le x \le 1-\theta; \quad 0 < \theta < 1$$

(i) Find the mean of the above distribution.

(ii) Derive Maximum likelihood estimators for θ and for mean of the above distribution.

(iii) A random sample drawn from the above distribution is given in the following table.

0.127	0.196	0.279	0.576	0.46
0.054	0.695	0.322	0.48	0.4
0.657	0.188	0.003	0.062	0.198
0.612	0.295	0.125	0.675	0.577

Estimate θ and mean of the above distribution using maximum likelihood estimators derived in part(ii).