The Open University of Sri Lanka

B.Sc/B.Ed. Degree Programme – Level 04

Open Book Test – 2023/2024

Pure Mathematics

PEU4300 – Real Analysis 1

Duration: - One Hour.

Date: - 29.07.2023 **Time: - From 10.30 a.m. To 11.30a.m.**

Answer All Questions

01) (a) Let $x_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}$. Prove that

(i) $\langle x_n \rangle$ is monotonically increasing

(ii) $n! \geq 2^{n-1}$ for each $n \in \mathbb{N}$

(iii) $\langle x_n \rangle$ is bounded above by 3.

(b) Prove that $\lim_{n\to\infty} \frac{3n-1}{4n+5} = \frac{3}{4}$, using the definition of limit.

(02) (a) Prove that $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$. Deduce that $\lim_{n\to\infty} \frac{1}{\sqrt{n+7}} = 0$.

(b) Let $\langle x_n \rangle$ be a sequence such that for each $n \in \mathbb{N}$,

$$\frac{1}{\sqrt{n+7}} \leq x_n - 2 \leq \frac{1}{\sqrt{n}}$$

Find $\lim_{n\to\infty} x_n$.

(c) Using the definition, show that the sequence $\langle 5 + (-1)^n \rangle$ diverges.