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The Open University of Sri Lanka
Department of Electrical and Computer Engineering
ECX6242 – Modern Control Systems
Final Examination – 2015/2016

Date: 2016-12-05 Time:0930-1230

Answer **five** questions by selecting **at least two questions** from each of the **sections A** and **B.**

Section A

Q1. A system is represented by the differential equation

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = \frac{du}{dt} + u$$

Where $y = output$ and $u = input$.

(a) Describe what are the advantages of state space modelling of a control system?

(b) Define the states as $x_1 = y$ and $x_2 = \frac{dy}{dt} - u$ and determine whether the system is controllable.

Q2. Consider the system represented in state variable form

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u$$
$$y = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}u$$

where

$\boldsymbol{A} = \begin{bmatrix} 1 & 1 \\ -5 & -10 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}, \boldsymbol{C} = [6 \quad -4]$ and $\boldsymbol{D} = [0]$.

(a) Verify that the system is observable and controllable.

(b) If so, design a full-state feedback law and an observer by placing the closed-loop system poles at $s_{1,2} = -1 \pm j$ and the observer poles at $s_{1,2} = -10$.

Q3.

(a) Briefly describe Lyapunov's direct method for the determination of the stability of non-linear systems.

(b) Consider the scalar system

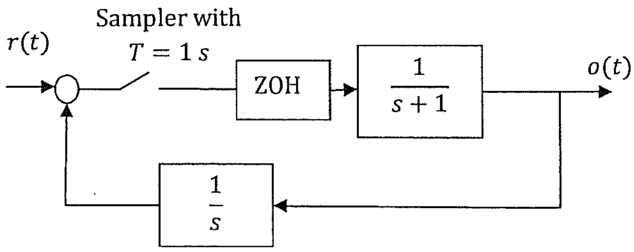
$$\dot{x} = ax^3$$

(i) Show that Lyapunov's linearization method fails to determine the stability of the origin.

(ii) Use Lyapunov's function $V(x) = x^4$ to show that the system is stable for $a < 0$ and unstable for $a > 0$.

(iii) What can you say about the system stability for $a = 0$?

Q4.
(a) Explain Sample and Hold (SOH) as applied to discrete systems.
(b) Determine the output in discrete form when a unit step is applied to the input of the following closed-loop system. (z-Transform pairs given at the end of the paper)



Section B

The questions in this section are based on the paper reproduced at the end of this question paper and your knowledge on control systems. Devote at least half an hour to reading through the paper. Use your own words in your answers so as to demonstrate that you have understood the concepts described in the paper, do not copy extracts from the paper itself.

Q5. Explain the structure of a regular PID controller and a tuning method.
Q6. Briefly describe the problems with multivariable control systems.
Q7. Briefly explain the proposed methodology in the paper.
Q8. Comment on the results obtained by the proposed method.

Note:

Laplace transform	Corresponding z-transform
$\frac{1}{s}$	$\frac{z}{z-1}$
$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$
$\frac{1}{s^3}$	$\frac{T^2 z(z+1)}{2(z-1)^3}$
$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$
$\frac{1}{(s+a)^2}$	$\frac{Tze^{-aT}}{(z-e^{-aT})^2}$
$\frac{a}{s(s+a)}$	$\frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$
$\frac{b-a}{(s+a)(s+b)}$	$\frac{z(e^{-aT}-e^{-bT})}{(z-e^{-aT})(z-e^{-bT})}$
$\frac{(b-a)s}{(s+a)(s+b)}$	$\frac{(b-a)z^2-(be^{-aT}-ae^{-bT})z}{(z-e^{-aT})(z-e^{-bT})}$
$\frac{a}{s^2+a^2}$	$\frac{z \sin aT}{z^2-2z\cos aT+1}$
$\frac{s}{s^2+a^2}$	$\frac{z^2-z\cos aT}{z^2-2z\cos aT+1}$
$\frac{s}{(s+a)^2}$	$\frac{z[z-e^{-aT}(1+aT)]}{(z-e^{-aT})^2}$

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Multivariable PID Controllers for Dynamic Process

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Abstract **This paper is concerned with the design of a dynamic multivariable PID control for multi input multi output (MIMO) process. Four multivariable PID control schemes using Davison, Penttinen-Koivo, Maciejowski and a combined method were applied. The controller parameters for all control strategies were designed based on dynamic condition using singularly perturbed system. The purpose of the study is to investigate the effectiveness in the performance of dynamic control based on different multivariable PID control strategies. To attain the best result, numerous tuning parameters were tested. The simulation results show the significance of the study whereby the proposed dynamic MPID control scheme shows better improvement in control tuning of nonlinear system.**

Keywords ***multivariable PID; dynamic process; nonlinear system; control tuning***

I. INTRODUCTION

Numerous real physical systems are based on multivariable process. The process causes an interaction to occur between the loops and makes the control design become tedious. Multivariable process can be controlled either by using decentralized or centralized controller. Decentralized controller involves several loops while centralized controller only involves one loop. Fig.1 shows a decentralized controller for two inputs two outputs system, whereas Fig. 2 shows a centralized controller for multivariable system.

Previously, [1 7] applied decentralized controller to the systems. Decentralized controller requires multiple single input single output (SISO) controllers where it is challenging compared to the single loop controller[1], [8]. Moreover, decentralized controller is also not capable or sometimes will fail to control the system with loop interactions[9]. Decentralized controller only gives a good performance if the loops interactions are modest. Therefore, to deal with a severe loops interaction a centralized controller was desired[10].

In this work, a method for controlling dynamic multivariable process is presented. Four methods were carried out to design a centralized controller by the Davison, Penttinen-Koivo, Maciejowski and combined method by[11]. Previously, [11], [12] used those techniques where it is only able to control the system at steady state. Here, those techniques underwent modification where the inverse of the process transfer function was modified. With that, the controller is capable of controlling the dynamic process.

The paper is organized as follows: Section II describes the model of wastewater treatment plant. In section III, the multivariable PID control strategies are described. Section IV deals with the simulation result and discussion. Finally, section V presents the conclusions.

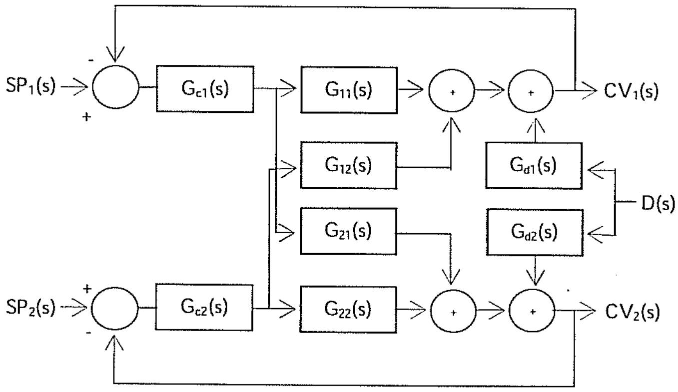

Fig. 1.Decentralized controller for two inputs two outputs system.

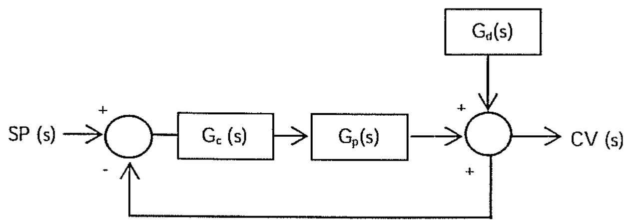

Fig. 2. Centralized controller for multivariable system.

II. MODEL FOR CONTROLLER DESIGN

MIMO system for wastewater treatment plant was described by a transfer function matrix, as shown in (1),

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$$Y(s) = G(s)U(s) \quad (1)$$

where *Y(s)*, *G(s)* and *U(s)* represent the output, process plant and input respectively. In order to verify the effectiveness of the centralized multivariable PID control, we address a singularly perturbed system of wastewater treatment plant as shown in Fig. 3. It comprised of an aerated tank and clarifier. There are four inputs and four outputs. However, we only consider two inputs and two outputs: dilution rate, air flow rate, substrate and dissolved oxygen.

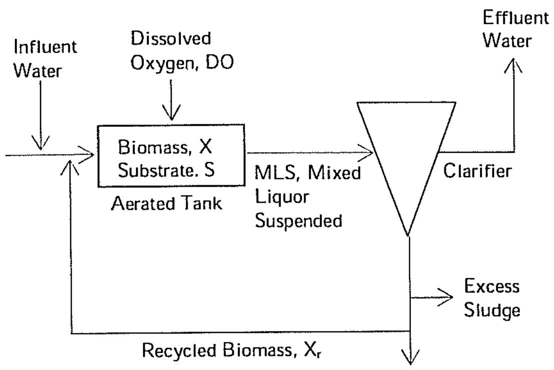


Fig. 3. Wastewater treatment plant.

The four nonlinear differential equations which describe the behavior of the wastewater treatment plant are expressed as (2) until (5) where the state variables *X(t)*, *S(t)*, *C(t)* and $X_r(t)$ represent the concentrations of biomass, substrate, dissolved oxygen and recycled biomass. *D(t)*, $\mu(t)$, $S_{in}(t)$, $C_{in}(t)$, C_s, *Y*, K_o, K_{La}, *r* and β represent dilution rate, specific growth rate, substrate concentrations of influent steams, dissolved oxygen concentration of influent steams, constant of maximum dissolved oxygen, rate of microorganism growth, model constant, constant of oxygen transfer rate coefficient, ratio of recycled and ratio of waste flow to the influent flow rate respectively.

$$\dot{X}(t) = \mu(t)X(t) - D(t)(1+r)X(t) + rD(t)X_r(t) \quad (2)$$

$$\dot{S}(t) = -\frac{\mu(t)}{Y}X(t) - D(t)(1+r)S(t) + D(t)S_{in} \quad (3)$$

$$\dot{C}(t) = -\frac{K_o\mu(t)}{Y}X(t) - D(t)(1+r)C(t) + K_{La}\big(C_s - C(t)\big) + D(t)C_{in} \quad (4)$$

$$\dot{X}_r(t) = D(t)(1+r)X(t) - D(t)(\beta + r)X_r(t) \quad (5)$$

III. Multivariable PID Control Strategies

A. Davison Method

The method proposed by Davison only applied integral term which causes decoupling to rise at a low frequency. The controller expression is quantified by (6).

$$\underline{u}(s) = k_i\frac{1}{s}\underline{e}(s) \quad (6)$$

Where k_i is the integral feedback gain which is expressed in (7)

$$k_i = \varepsilon G^{-1}(0) \quad (7)$$

ε is the tuning parameter, it can be tuned instantaneously until the greatest solution is attained, while $G(0)$ is the process transfer function matrix and $\underline{e}(s)$ is the controller error.

In this work, the expression for integral feedback gain was modified to $k_i = \varepsilon G^{-1}(s)$ where it is able to determine the dynamic process control.

B. Penttinen-Koivo Method

Penttinen-Koivo method applied both integral and proportional term. This method gives a bit of improvement than the method proposed by Davison where good decoupling characteristic occur at both low and high frequency. The controller expression is quantified by (8).

$$\underline{u}(s) = \left(k_p + k_i\frac{1}{s}\right)\underline{e}(s) \quad (8)$$

Where k_i is the integral feedback gain which expresses as in (9) and k_p is the proportional gain which expresses as in (10).

$$k_i = \varepsilon G^{-1}(0) \quad (9)$$

$$k_p = (CB)^{-1}\rho \quad (10)$$

ε and ρ is the tuning parameter and $\underline{e}(s)$ is the controller error. Alike to Davison method, to determine the dynamic process control, integral feedback gain was modified to $k_i = \varepsilon G^{-1}(s)$.

C. Maciejowski Method

Using Maciejowski method, the system is diagnolised close to the bandwidth frequency, w_B. It combines all integral, proportional and derivative term together. The controller expression is quantified by (11).

$$K = \left(K_p + K_i\frac{1}{s} + K_d s\right) \quad (11)$$

Where K_p, K_i and K_d is the proportional, integral and derivative gain which is defined as (12) until (14). ρ,ε and δ are scalar tuning parameters.

$$K_p = \rho G^{-1}(jw_b) \quad (12)$$

$$K_i = \varepsilon G^{-1}(jw_b) \quad (13)$$

$$K_d = \delta G^{-1}(jw_b) \quad (14)$$

D. Combined Method

The controller expression for the combine method is expressed in (15) until (17).

$$K_p = \rho G^{-1}(jw_b) \qquad (15)$$

$$K_i = \varepsilon G^{-1}(0) \qquad (16)$$

$$K_d = \delta(CB)^{-1} \qquad (17)$$

This technique was proposed to meet the industry s needs. Previously, Maciejowski method requires plant frequency analysis experiments where it is difficult to satisfy the industry s need. Therefore, this method introduced new approach to eliminate the need of frequency analysis [11].

IV. RESULT AND DISCUSSION

To show the performance of the centralized multivariable PID in controlling the concentration of substrate and dissolved oxygen of a wastewater treatment system, simulation studies are presented in this section. The simulations are done by using Matlab/ Simulink software. For each simulation run, the substrate and dissolved oxygen were injected with step response at 10h. Consider (18) until (21) is the transfer function matrix for the linear multivariable system.

$$G_{11}(s) = \frac{134s^2 + 273.1s + 8.869}{s^3 + 2.297s^2 + 0.6071s + 0.01195} \qquad (18)$$

$$G_{12}(s) = \frac{-0.03117s - 0.001029}{s^3 + 2.297s^2 + 0.6071s + 0.01195} \qquad (19)$$

$$G_{21}(s) = \frac{-9.083s^2 - 13.43s - 0.1847}{s^3 + 2.297s^2 + 0.6071s + 0.01195} \qquad (20)$$

$$G_{22}(s) = \frac{0.06994s^2 + 0.02251s + 0.0004382}{s^3 + 2.297s^2 + 0.6071s + 0.01195} \qquad (21)$$

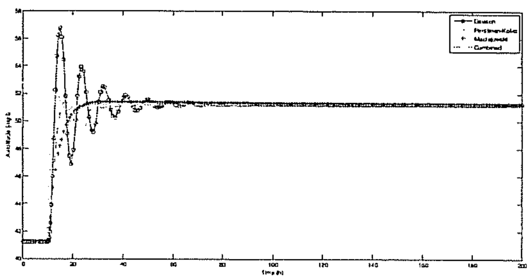
The coefficients of the tuning parameter are varied according to the behavior of the responses, until the desired responses are obtained. Table 1 shows the tuning parameter of ρ, ε, δwhich is used to define K_p, K_i, and K_d parameters in (7), (9), (10), and (12) until (17). Table 1 also shows the performance characteristic of the rise time, T_R, settling time, $T_s$and percentage overshoot, %OS for each control strategies.

TABLE I. TABLE STYLES

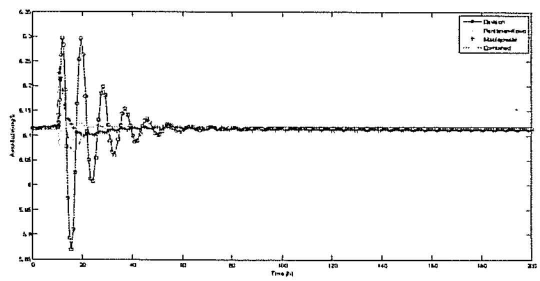
Control Strategies	Tuning Parameter			Performance Characteristic			
	ρ	*ε*	*δ*	*W_b*	*T_R(h)*	*T_s(h)*	*% OS*
Davison	0	1.000	0	-	1.5	54.6	0.1067
Penttinen-Koivo	1	1.125	0	-	2.0	20.0	0.0167
Maciejowski	1	0.312	0	0.05	8.5	23.0	0.0026
Combined	1	0.357	0	0.05	10.5	36.0	0.0000

The resulting responses obtained during substrate change are shown in Fig. 4. Fig. 4a shows the concentration of substrate. Based on the responses obtained, Davison method produced an enormous overshoots which is typically not desired at all. Penttinen-Koivo method produces a response with less oscillatory than Davison method and provided rapid settling time, whereas response by Maciejowski method provides better settling time than Davison method. However the response is faced with time consuming problem, where it required long time to rise from 10% to 90% of its final value. Response corresponding to the Combined method gives a good response in terms of percentage overshoot, where no overshoot was observed. Nonetheless, the response is slow in terms of rise and settling time. However, the settling time is still better than Davison method.

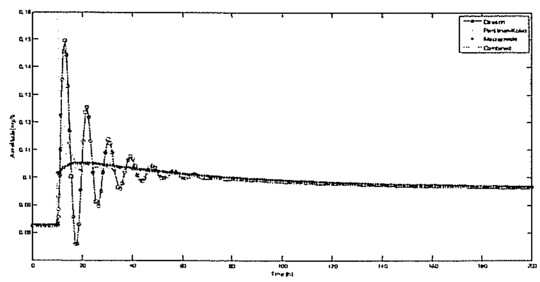
In multivariable system, each manipulated variable may affect several controlled variables. This causes multivariable control to be more difficult to control compared to the single input single output system. Fig. 4b shows the interactions that occur for dissolved oxygen during substrate change. From the responses, it shows that all applied control strategies are able to overcome or handle interaction that occur between the input/ output loops.



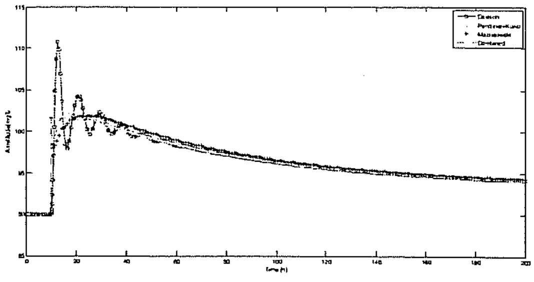
a. Concentration of substrate.



b. Concentration of dissolved oxygen.



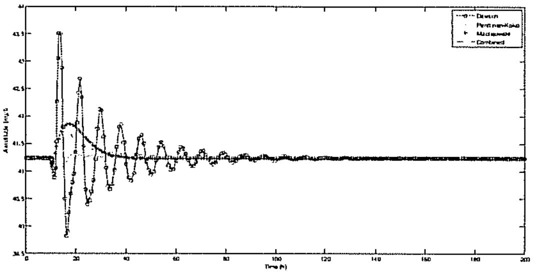
c. Concentration of dilution rate



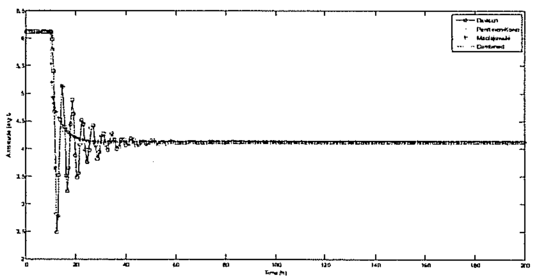
d. Concentration of air flow rate.

Fig.4. Responses during substrate change.

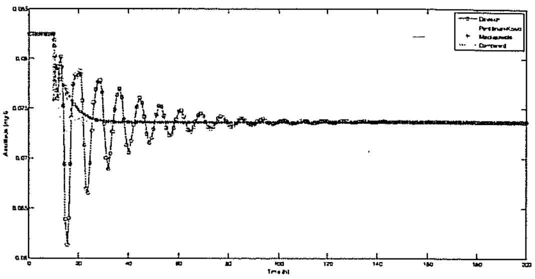
Fig. 5 shows the response graph which represents the concentration of substrate, dissolved oxygen, dilution rate and air flow rate during dissolved oxygen change for each control strategies. Fig. 5a, 5c and 5d present the interactions between inputs outputs of the system. Based on the author s point of view and also referring to Fig. 5b, the applied controller is able to meet the objective control where the concentration of dissolved oxygen settled down at 4.234 mg/l. Based on the four applied methods, Davison method gave the poorest control performance compared to the others due to the high oscillation. Meanwhile, Combined method provides the best control capability with no oscillation and rapid time for dynamic response to reach at 4.234 mg/ l.



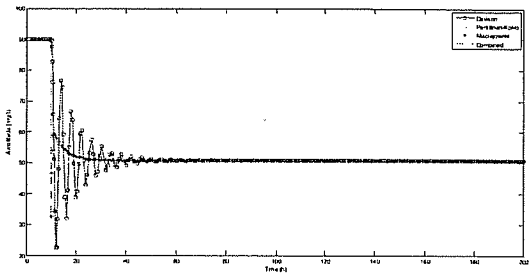
a. Concentration of substrate.



b. Concentration of dissolved oxygen.



c. Concentration of dilution rate.



d. Concentration of air flow rate.

Fig.5. Responses during dissolved oxygen change.

V. CONCLUSION

In this paper, four multivariable control strategies for dynamic process are present, where the control strategies are based on simple PID controller. From extensive simulation studies carried out on a nonlinear model, it is demonstrated that the dynamic response of MPID control based on Davison, Penttinen-Koivo, Maciejowski and Combined methods produced sensible results. However, owing to the characteristic which only involve integral term, Davison method produce high overshoots. While other methods show better results with respect to decoupling capabilities and closed-loop performance.

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