The Open University of Sri Lanka
B.Sc./ B.Ed. Degree Programme
Pure Mathematics – Level 05
PEU5302 – Combinatorics
Open Book Test (OBT) – 2023/2024

Duration: One (01) hour

Date: 28.07.2023 Time: 9.00 a.m. -10.00 a.m.

ANSWER ALL QUESTIONS.

(01) Let f be the function from $X = \{1,2,3,4\}$ to $Y = \{a, b, c, d, e\}$. Find the number of functions that can be formed from X to Y. How many of those functions are injective?

(02) (a) Sarah writes down random positive integers when she gets bored. Prove that if Sarah writes 1001 numbers, then there must be at least 2 with the same last three digits.

(b) Using the Pigeonhole Principle, show that among any 4 numbers one can find 2 numbers with their difference is divisible by 3. (Avoid considering the cases separately.)

(03) (a) A cricket team consists of 16 players. It includes 2 wicket keepers and 5 bowlers. In how many ways can a team with eleven players be selected if the coach has to select 1 wicket keeper and at least 4 bowlers?

(b) The Department of Applied Mathematics has nineteen faculty members, of whom three are women. How many committees of four can be formed if

(i) the head of the department is not eligible to be in the committee?
(ii) exactly one of the members must be a woman?

(iii) exactly one must be a women, and two men professors Smith and Jones refuse to serve together?

(04) (a) (i) How many integers greater than ten thousand can be formed from the digits 0, 1,2,3 and 4 without repetition?

(ii) In how many ways can 4 boys and 4 girls be seated around a circular table so that no two boys are in adjacent positions?

(b) Eight chairs are numbered from 1 to 8. Two women and three men wish to occupy one chair each. First, the women choose the chairs amongst the chairs marked 1 to 4, and then the men select the chairs amongst the remaining. Calculate the number of possible choices?

(05) In how many ways can the alphabets of the word " EXCELLENT " be arranged?

(06) Let r be an integer. It is defined that ${}^{n}P_{r}$ be the number of ways of arranging r places from n different items. If ${}^{56}P_{r+6} : {}^{54}P_{r+3} = \mathbf{30800:1}$, find ${}^{r}P_{2}$