The Open University of Sri Lanka
Department of Electrical and Computer Engineering
ECX6241 – Field Theory
Final Examination – 2015/2016

Date: 2016-11-25 Time: 0930-1230

Instructions for Candidates:

- Answer **five questions** by selecting **two questions** from **Section A, two questions** from **Section B** and **one question** from **Section C.**
- Each question of **Section A, B** and **C** carries **15, 20** and **30** marks respectively.
- All the notations have their usual meanings.

Section A

Q1.

(a) Explain the physical meaning of the divergence of a vector. [4]

(b) Prove that $\nabla \cdot \boldsymbol{r} = 3$ and $\nabla \times \boldsymbol{r} = 0$ if $\boldsymbol{r} = x\,\boldsymbol{a}_x + y\,\boldsymbol{a}_y + z\,\boldsymbol{a}_z$ [4]

(c) Find the rate at which the scalar function $V = r^2 \sin 2\phi$ in cylindrical coordinates in the direction of the vector $\boldsymbol{A} = \boldsymbol{a}_r + \boldsymbol{a}_\phi$ at the point $\left(2, {}^{\pi}/_{4}, 0\right)$ [7]

Q2.

(a) Show that $\nabla^2 \boldsymbol{F} = \nabla(\nabla \cdot \boldsymbol{F}) - \nabla \times \nabla \times \boldsymbol{F}$ [3]

(b) Find the Laplacian of the scalar field

$$F = rz \sin\phi + r^2 + z^2 \cos\phi$$ [5]

(c) Prove that $\nabla^2 \left(\frac{1}{r}\right) = 0$, with usual meaning for r. [7]

Q3.

(a) State the Divergence theorem [3]

(b) Discuss the properties of a solenoidal and rotational vector fields. [4]

(c) Given that the vector $\boldsymbol{D} = {}^{5\rho^2}/_{4}\,\boldsymbol{a}_\rho$ in spherical coordinates. Verify both sides of the divergence theorem for the volume enclosed by $\rho = 1$ and $\rho = 2$. [8]

Section B

Q4.

(a) Discuss three properties of electric flux. [3]

(b) A line charge of length of $2l$ has a linear charge density of λ. Show that the electric field at a distance r is

$$\boldsymbol{E}_r = {}^{\lambda}\Big/_{2\pi\varepsilon_0 r\sqrt{\left(\frac{r}{l}\right)^2 + 1}}\,\boldsymbol{a}_r$$ [12]

(c) Find and plot $\lim_{l\to\infty} \boldsymbol{E}_r$ [3]

(d) Comment on the result in (c). [2]

Q5.

(a) What is meant by the edge effect in capacitors? [3]

(b) A capacitor is made of two coaxial metallic cylinders of radii r_1 and r_2 ($r_1 < r_2$) and length L ($L >> r_2$). The region between r_1 and $r_3 = \sqrt{r_1 r_2}$ is filled with a medium of dielectric constant K_1 and the remaining region is filled with a medium of dielectric constant K_2. Find the capacitance of the system. [12]

(c) Hence, derive the equation for the capacitance of a coaxial capacitor. [5]

Q6.

(a) Draw the Hysteresis curve or magnetisation (B-H) curve and explain the different regions. [6]

(b) Show that the self-inductance of a solenoid of finite length l, radius r and N number of turns is

$$L = \frac{\mu N^2 \pi r^2}{l^2}\left(\sqrt{r^2 + l^2} - r\right) \quad [10]$$

(c) Hence find the self –inductance per unit length of an infinitely long solenoid. [4]

Section C

Q7.

(a) Describe and discuss the medium characteristics of

i. Free space
ii. Lossless dielectrics
iii. Lossy dielectrics
iv. Good conductors [2×4]

(b) State the Poynting's theorem and discuss its physical interpretation. [6]

(c) A medium characterized by $\sigma = 0, \mu = \mu_0, \varepsilon = \varepsilon_0$ and $\boldsymbol{E} = 20\sin(10^8 t - \beta z)\,\boldsymbol{a}_y$

i. Show that $\boldsymbol{E}$ satisfies the Gauss's law for electric fields. [4]
ii. Calculate β and $\boldsymbol{H}$. [12]

Q8. Briefly explain any three topics of the following.

(a) Micro strip patch antenna
(b) Skin depth
(c) Electromagnetic induction
(d) Magnetic resonance imaging
(e) Magnetic levitation [10 × 3]